Intermediate Math Circles

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STATE MACHINES

The Centre for Education in Mathematics and Computing
Faculty of Mathematics, University of Waterloo

www.cemc.uwaterloo.ca

A Game

A game involves sailing between islands in order to find buried treasure. There are four islands named Ogden, Higgins, Gillies, and Meyers. From each island you can choose to sail on either the red ship or the blue ship. The colour of ship determines your destination as follows:

- From Ogden, taking the red ship brings you to Meyers and taking the blue ship brings you to Higgins.

- From Higgins, taking the red ship brings you to Gillies and taking the blue ship brings you to Meyers.

- From Gillies, taking the red ship brings you to Meyers and taking the blue ship brings you to Ogden.

- From Meyers, taking the red ship brings you to Gillies and taking the blue ship gets you lost for a little while, eventually returning back to Meyers.

A Game

The game begins with you starting on Ogden. The treasure is buried on Gillies. Some questions you might ask are:

- Is it even possible to sail from Ogden to Gillies?

- How many routes from Ogden to Gillies are there?

- What is the shortest route from Ogden to Gillies? Is it unique?

A Model

Questions like these can be hard to answer without first playing the game, but as a game designer, it would be far better to answer these questions first. Imagine how much time, effort, and money would be lost if game designers built this game first and then realized that it is not even possible to win!
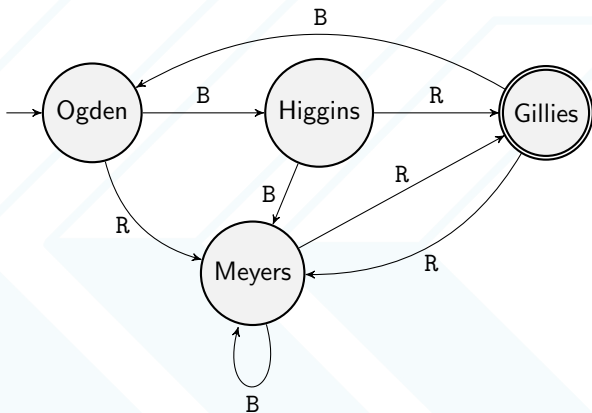
A compromise is to build a model of the game. A model allows you to simulate and analyze the game without actually having to build it.

(This idea works for other processes too, not just game design.)

A Model

Here is a model of our game. This type of model is called a state machine.

A Model

The circles (islands) are called states. The arrows between circles are called transitions and they describe how to change states. Changing states happens either by sailing on a red (R) ship or on a blue (B) ship. The arrow leading from "nowhere" to a circle indicates the starting state. In our game the starting state is Ogden island. A double circle indicates an accepting state. Accepting states are desired outcomes. In our game the desired outcome is Gillies island.

You can explore the model of our game using the following GeoGebra app:

geogebra.org/m/cpwtbyep

Problem Set

For each question, start the game on Ogden island.

1. Suppose you sail on 6 ships in the order B R B R B R. Which island are you now on?

2. Suppose you sail on 10 ships in the order B B R B R R B B R B. Which island are you now on?

3. Suppose you sail on 7 ships in the order R R B B ? B B. If you are now on Higgins island, what colour was the 5th ship you sailed on?

4. Suppose you sail on 12 ships. The last ship you sailed on was blue. Is it possible that you are now on Gillies island? Explain.

5. Suppose you sail on 9 ships. The last ship you sailed on was red. Is it possible that you are now on Gillies island? Explain.

6. Suppose you sail on 3 ships. Which island are you definitely *not* on now? How do you know?