

Intermediate Math Circles

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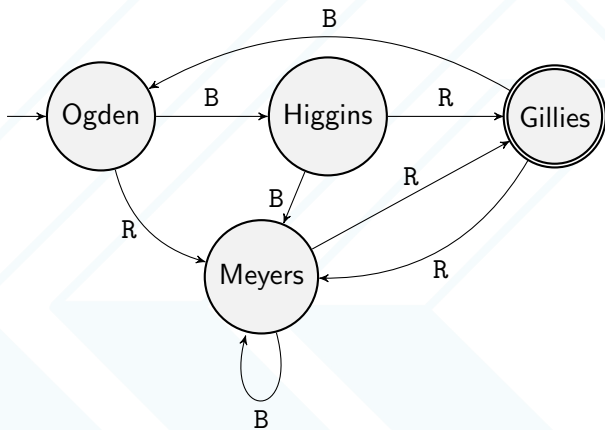
STATE MACHINES - SOLUTIONS

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Recall - Our Model



Problem Set - Solutions

1. Suppose you sail on 6 ships in the order B R B R B R. Which island are you now on?

Gillies.

2. Suppose you sail on 10 ships in the order B B R B R R B B R B. Which island are you now on?

Ogden.

3. Suppose you sail on 7 ships in the order R R B B ? B B. If you are now on Higgins island, what colour was the 5th ship you sailed on?
The first 4 ships take you to Higgins island. If you now take the blue ship (followed by two more blue ships) you will arrive at Meyers island. If you instead take the red ship (followed by two blue ships) you will arrive at Higgins island. Therefore, the 5th ship you sailed on was red.



Problem Set - Solutions

4. Suppose you sail on 12 ships. The last ship you sailed on was blue. Is it possible that you are now on Gillies island? Explain.
No. The only way to arrive at Gillies island is by a red ship.
5. Suppose you sail on 9 ships. The last ship you sailed on was red. Is it possible that you are now on Gillies island? Explain.
It is possible, but not guaranteed. If your first 8 ships were also red then you will now be on Meyers island. However, if your first 8 ships were blue, then yes, you will now be on Gillies island.
6. Suppose you sail on 3 ships. Which island are you definitely *not* on now? How do you know?
Higgins. If the third ship arrived at Higgins it must have come from Ogden. Therefore the second ship must have come from Gillies. Therefore the first ship must have come from Higgins or Meyers. But we know the first ship comes from Ogden. So it is not possible to be on Higgins island.

