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Centre for Education in
Mathematics and Computing

Grade 6 Math Circles

April 7th, 2021

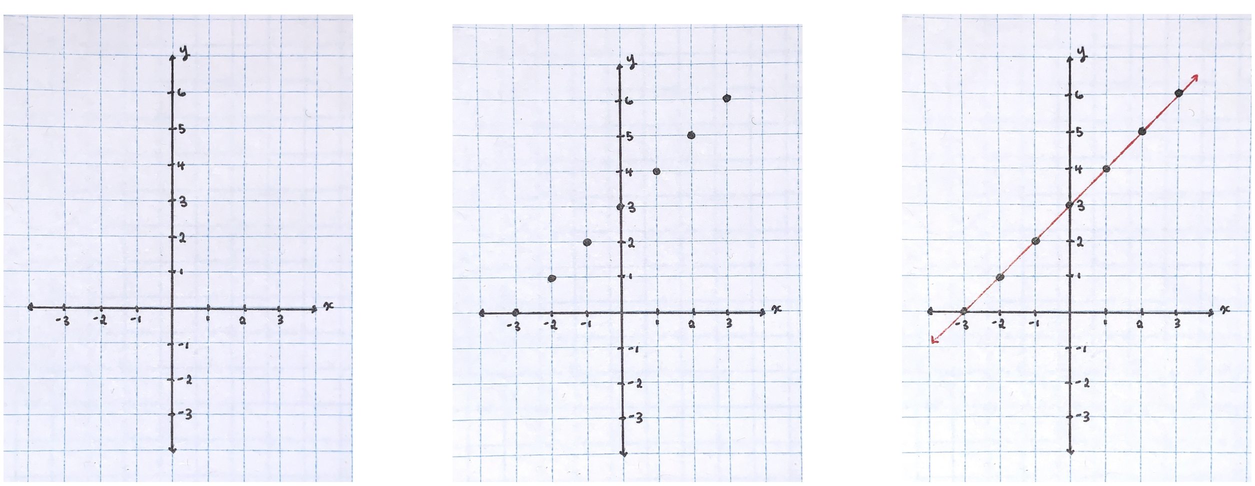
Graphing Functions: Problem Set Solutions

To graph functions, it's easiest to work by hand on paper (or on a tablet or touchscreen). You should draw and label the x and y axes, as well as mark down numbers along each axis (like on a number line). You can then plot points and graph functions on this Cartesian Plane. This will be the most straightforward with graph paper! At the end of this PDF is a sheet of graph paper which you can download to draw on or print out.

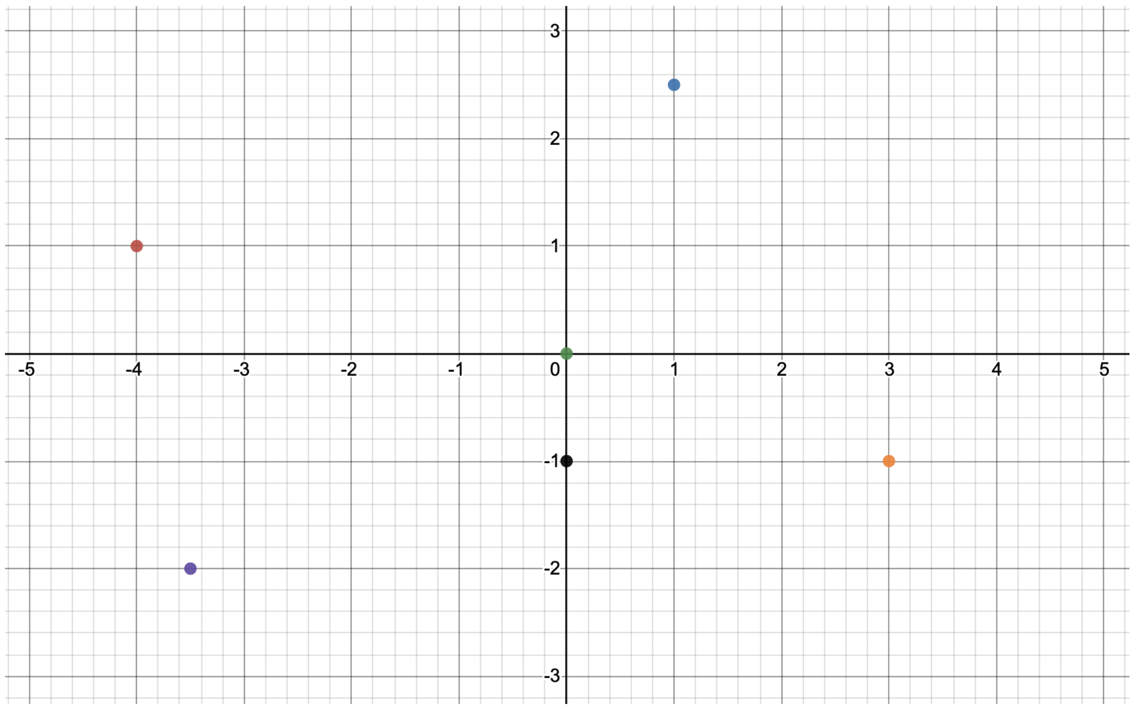
It is helpful to have a sense of what the x and y values will be before you begin graphing. Refer to this PDF: *https://www.cemc.uwaterloo.ca/events/mathcircles/2020-21/Winter/Junior6_Functions_Mar31_Solutions.pdf*, which contains tables for Questions 4–7 filled in for $x = -3$ to $x = 3$. You can make the graph as big or as small as you need to fit a reasonable amount of the function! Here is an example, drawn by hand on graph paper.

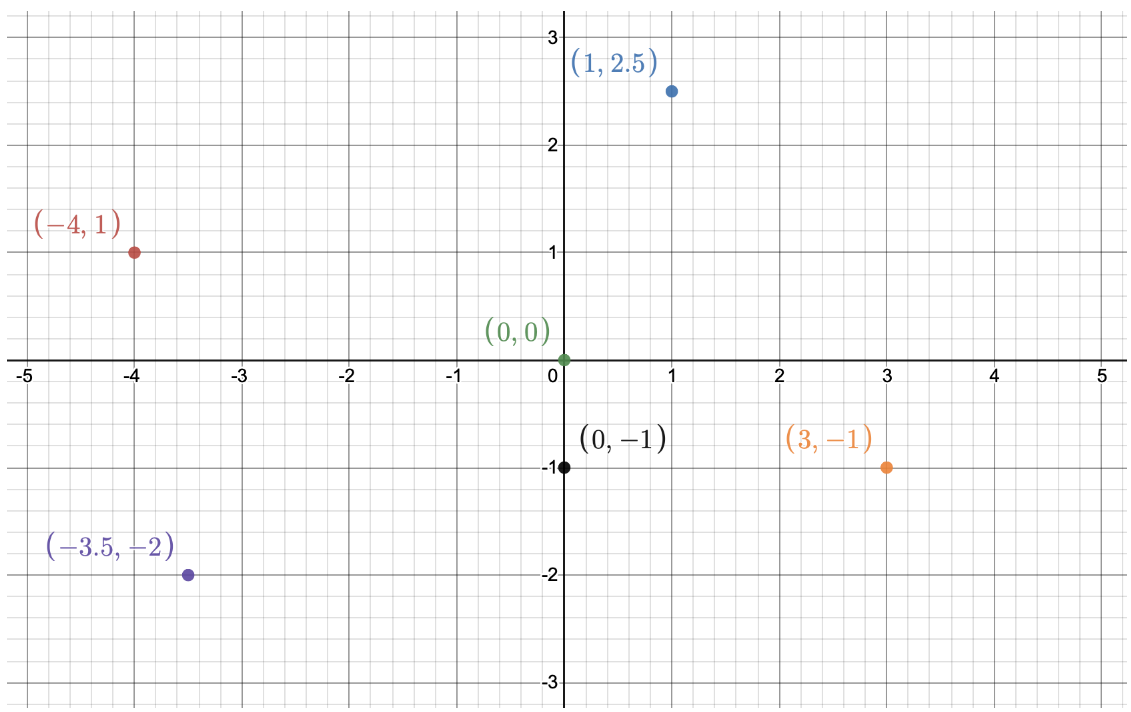
x	-3	-2	-1	0	1	2	3
$y = f(x) = x + 3$	0	1	2	3	4	5	6

With the help of this table, we draw the x- and y- axis of the Cartesian Plane to fit the points of our graph. Don't forget to label the axes! We can then add points at the coordinates we've calculated, which in this case are $(-3, 0)$, $(-2, 1)$, $(-1, 2)$, $(0, 3)$, $(1, 4)$, $(2, 5)$, and $(3, 6)$. Lastly, we connect our dots to reveal the graph of this linear equation!



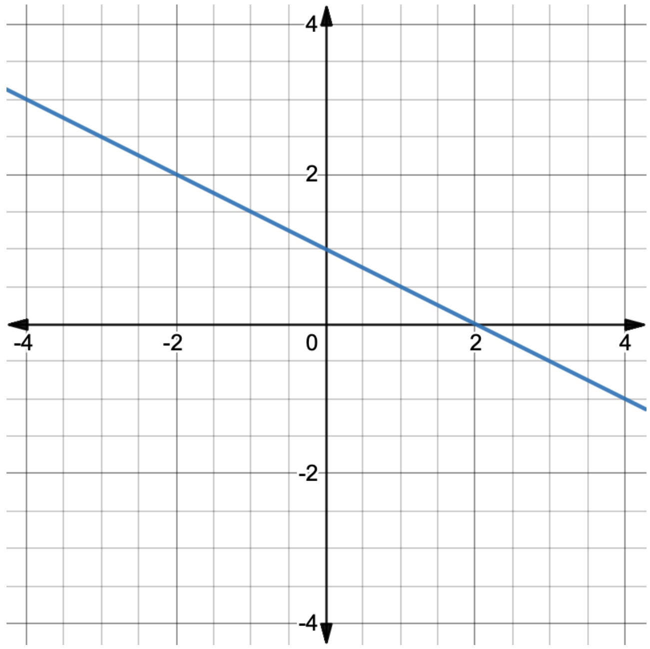
1. Identify the coordinates of the six points plotted below:





2. Use the graph to identify the value of y for each of the given values of x.

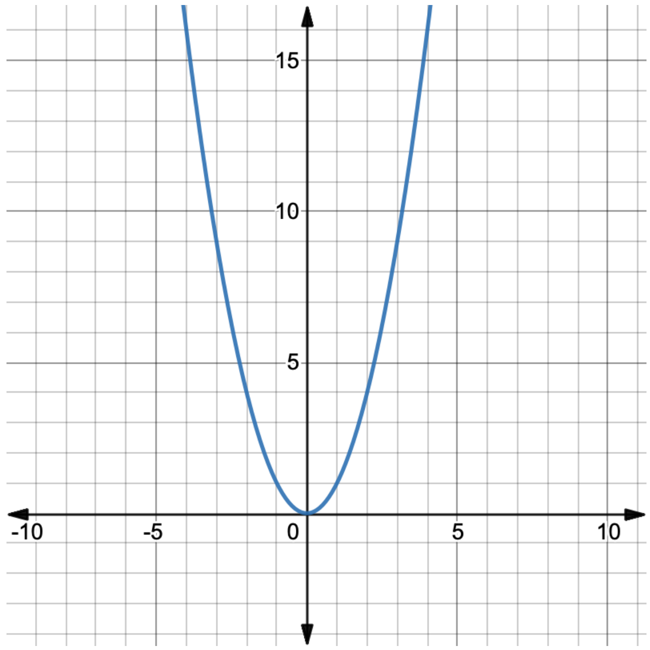
(a) What is y at $x = -3, -2, -1, 0, 1, 2, 3$ for this function?



Solution:

- at $x = -3$, $y = 2.5$
- at $x = -2$, $y = 2$
- at $x = -1$, $y = 1.5$
- at $x = 0$, $y = 1$
- at $x = 1$, $y = 0.5$
- at $x = 2$, $y = 0$
- at $x = 3$, $y = -0.5$

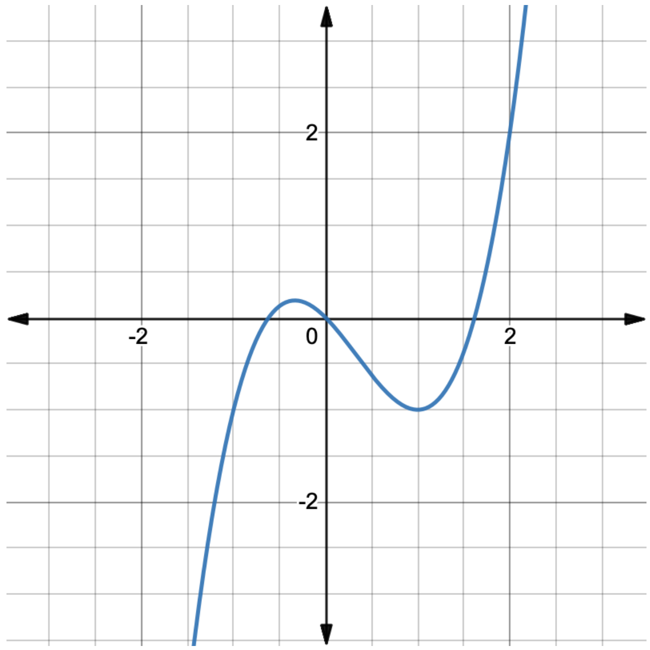
(b) What is y at $x = -4, -3, -2, -1, 0, 1, 2, 3, 4$ for this function?



Solution:

- at $x = -4$, $y = 16$
- at $x = -3$, $y = 9$
- at $x = -2$, $y = 4$
- at $x = -1$, $y = 1$
- at $x = 0$, $y = 0$
- at $x = 1$, $y = 1$
- at $x = 2$, $y = 4$
- at $x = 3$, $y = 9$
- at $x = 4$, $y = 16$

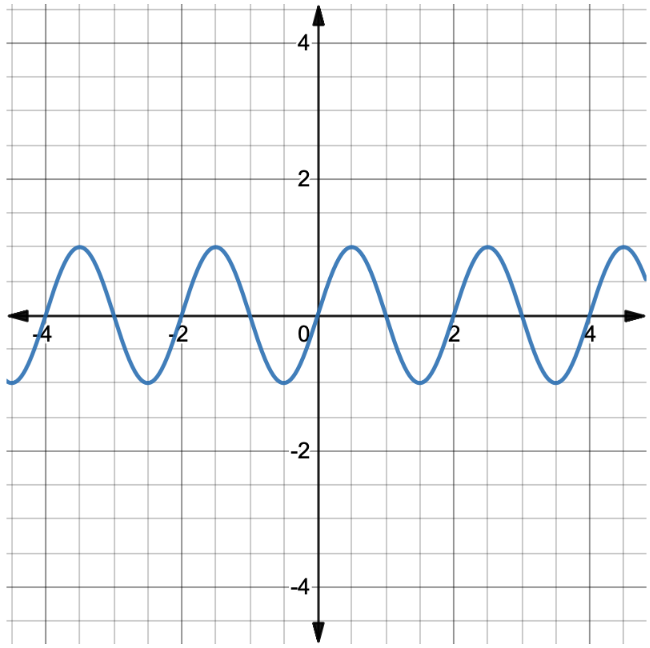
(c) What is y at $x = -1, 0, 1, 2$ for this function?



Solution:

- at $x = -1$, $y = -1$
- at $x = 0$, $y = 0$
- at $x = 1$, $y = -1$
- at $x = 2$, $y = 2$

3. At what values of x is the value of y equal to 1 for this function?



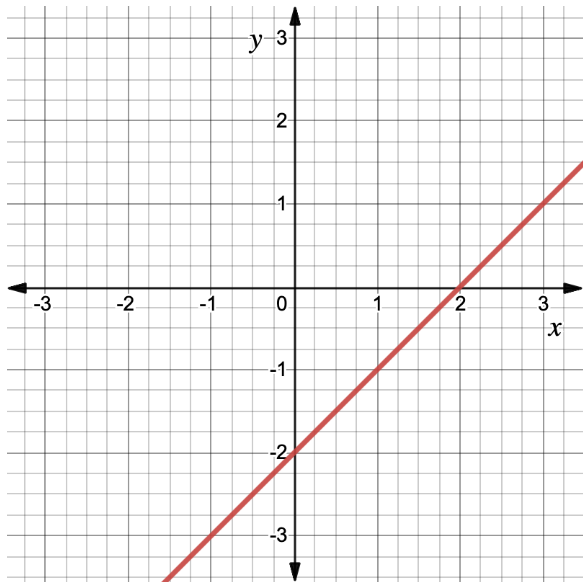
Solution: On the section of the function shown, $y = 1$ at $x = -3.5, -1.5, 0.5, 2.5$, and 4.5.

As a general formula, $y = 1$ at $x \in \{0.5 + 2n,\ n \in \mathbb{Z}\}$. That is, $y = 1$ at all values of x in the set of numbers that can be expressed as $0.5 + 2n$, when n is an integer.

4. Graph each function. Identify the domain and range.

(a) $f(x) = x - 2$

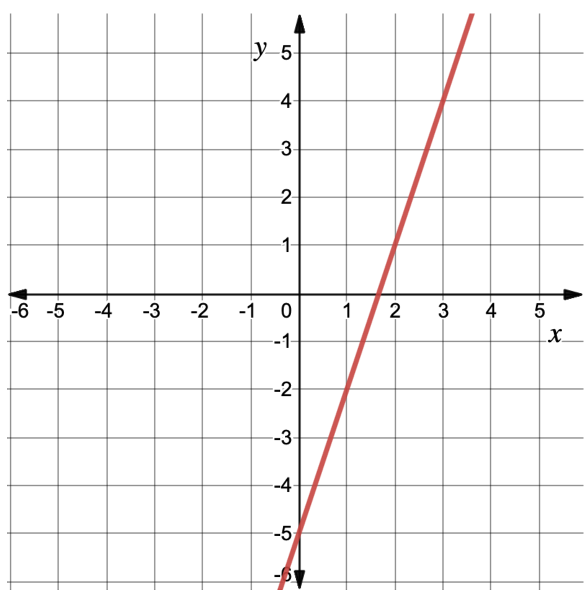
Solution: Domain: $\mathbb{R}$; Range: $\mathbb{R}$



https://www.desmos.com/calculator/axazyhah59

(b) $g(x) = 3x - 5$

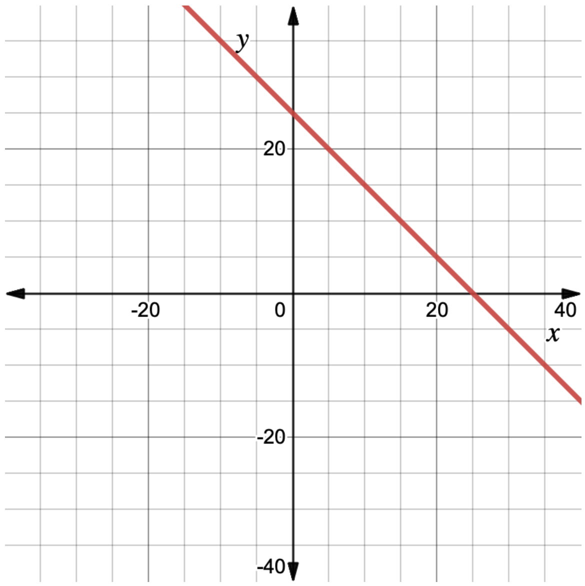
Solution: Domain: $\mathbb{R}$; Range: $\mathbb{R}$



https://www.desmos.com/calculator/buxqytfzvs

(c) $h(x) = -x + 25$

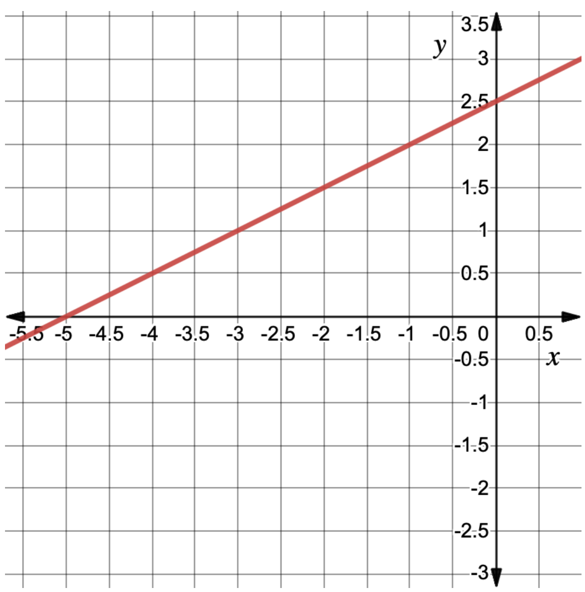
Solution: Domain: $\mathbb{R}$; Range: $\mathbb{R}$



https://www.desmos.com/calculator/5adif7nimr

(d) $j(x) = \frac{x+5}{2}$

Solution: Domain: $\mathbb{R}$; Range: $\mathbb{R}$



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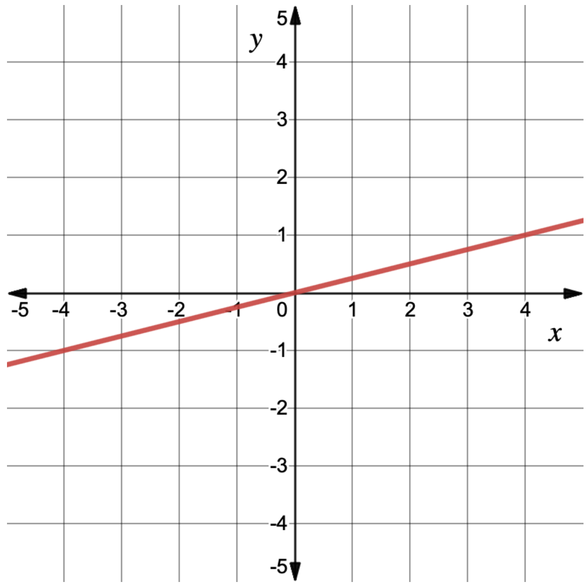
(e) $k(x) = \frac{1}{4}x$

Solution: Domain: $\mathbb{R}$; Range: $\mathbb{R}$



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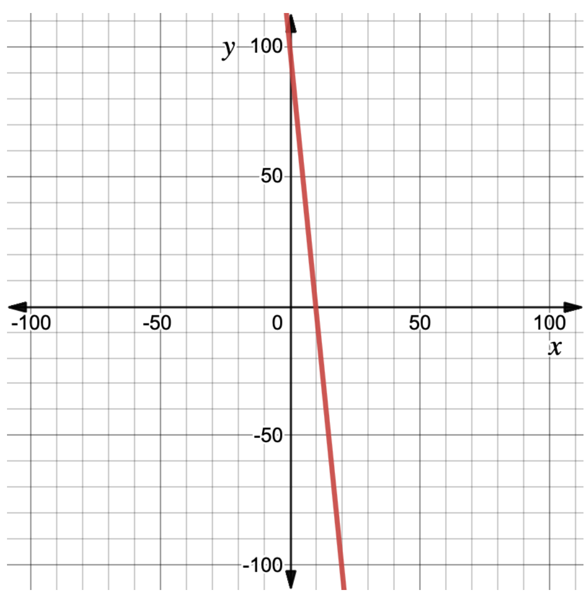
(f) $l(x) = 100 - 10x$

Solution: Domain: $\mathbb{R}$; Range: $\mathbb{R}$

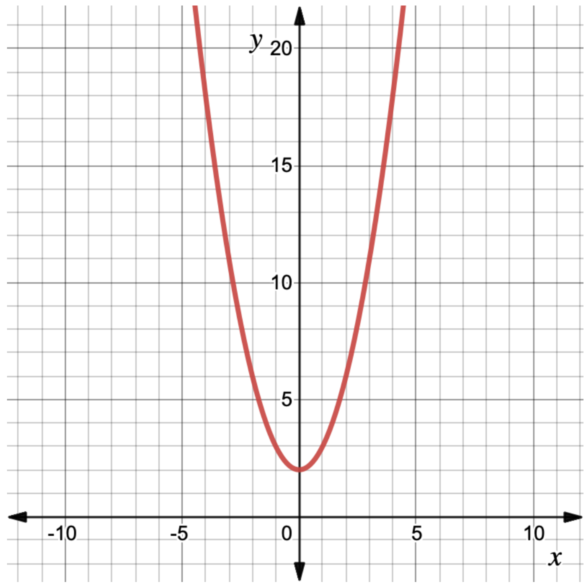


https://www.desmos.com/calculator/yuufikdwq9

5. Graph each function. Identify the domain and range.

(a) $f(x) = x^2 + 2$

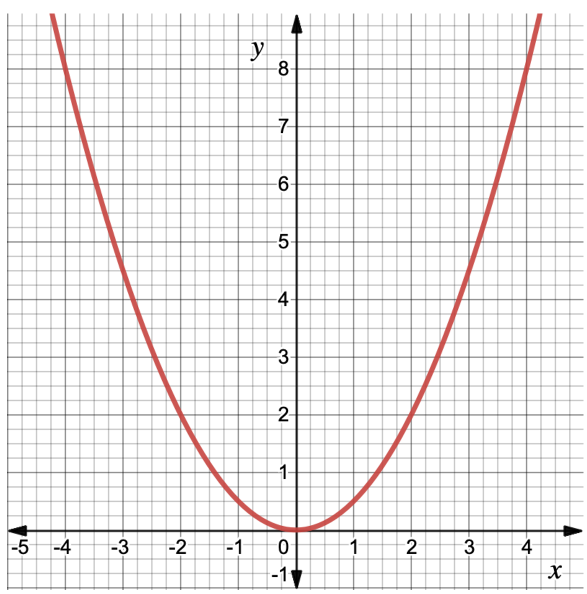
Solution: Domain: $\mathbb{R}$; Range: $\{y \in \mathbb{R},\, y \geq 2\}$



https://www.desmos.com/calculator/h9duqa1nre

(b) $g(x) = \frac{x^2}{2}$

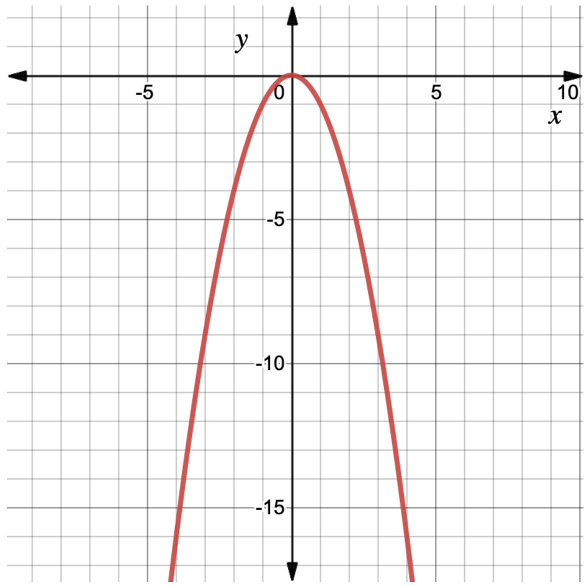
Solution: Domain: $\mathbb{R}$; Range: $\{y \in \mathbb{R},\, y \geq 0\}$



https://www.desmos.com/calculator/ekour66gqj

(c) $h(x) = -x^2$

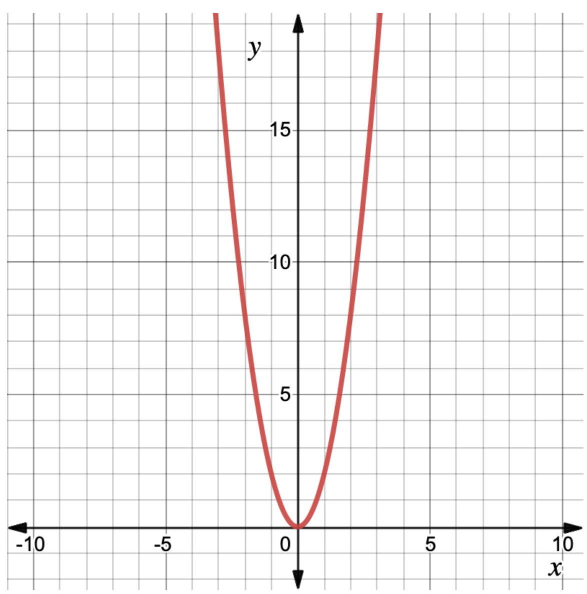
Solution: Domain: $\mathbb{R}$; Range: $\{y \in \mathbb{R},\, y \leq 0\}$



https://www.desmos.com/calculator/xw0wemlnh2

(d) $j(x) = 2x^2$

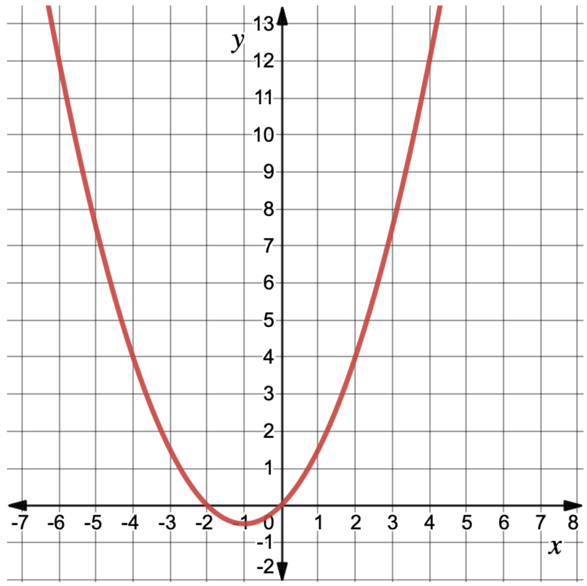
Solution: Domain: $\mathbb{R}$; Range: $\{y \in \mathbb{R},\, y \geq 0\}$



https://www.desmos.com/calculator/un5ww1gugm

(e) $k(x) = 0.5x^2 + x$

Solution: Domain: $\mathbb{R}$; Range: $\{y \in \mathbb{R},\ y \geq -0.5\}$



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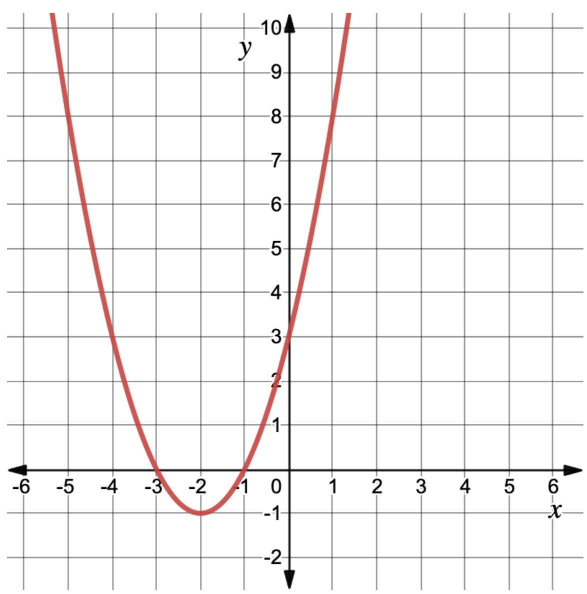
(f) $l(x) = x^2 + 4x + 3$

Solution: Domain: $\mathbb{R}$; Range: $\{y \in \mathbb{R},\ y \geq -1\}$

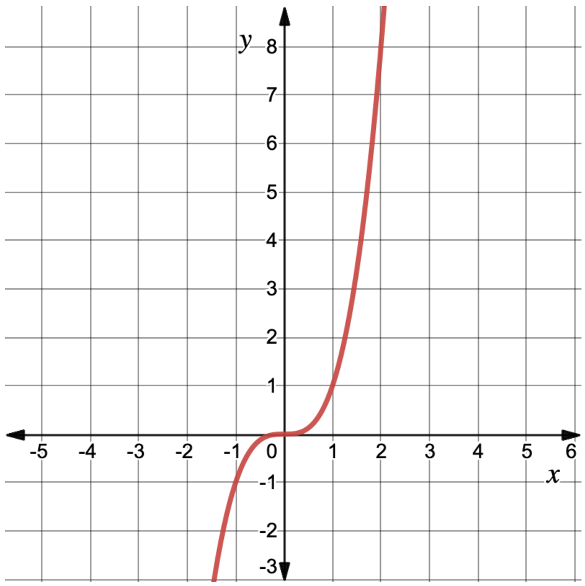


https://www.desmos.com/calculator/qgvj3o4igu

6. Graph each function. Identify the domain and range.

(a) $f(x) = x^3$

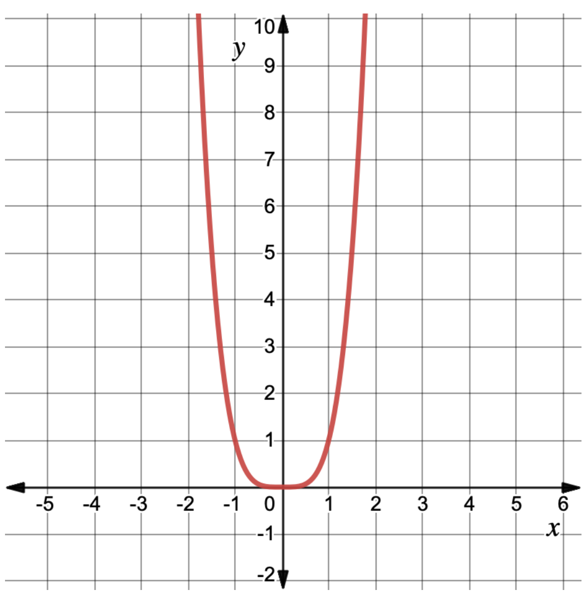
Solution: Domain: $\mathbb{R}$; Range: $\mathbb{R}$



https://www.desmos.com/calculator/8sji4e1xk8

(b) $g(x) = x^4$

Solution: Domain: $\mathbb{R}$; Range: $\{y \in \mathbb{R},\, y \geq 0\}$

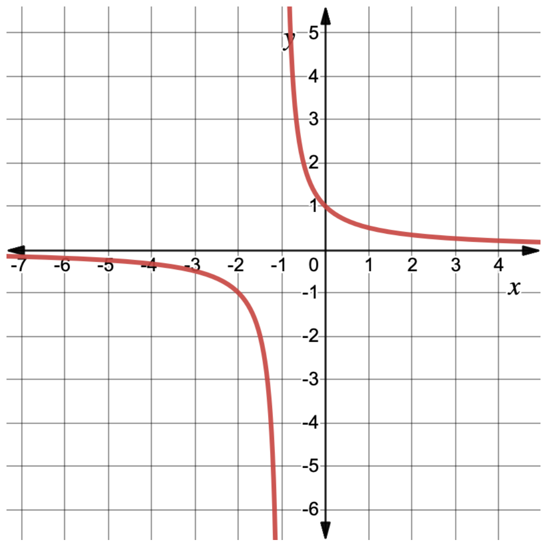


https://www.desmos.com/calculator/yuqr735dcl

7. Graph each function. Identify any asymptotes. Identify the domain and range.

(a) $f(x) = \frac{1}{x+1}$

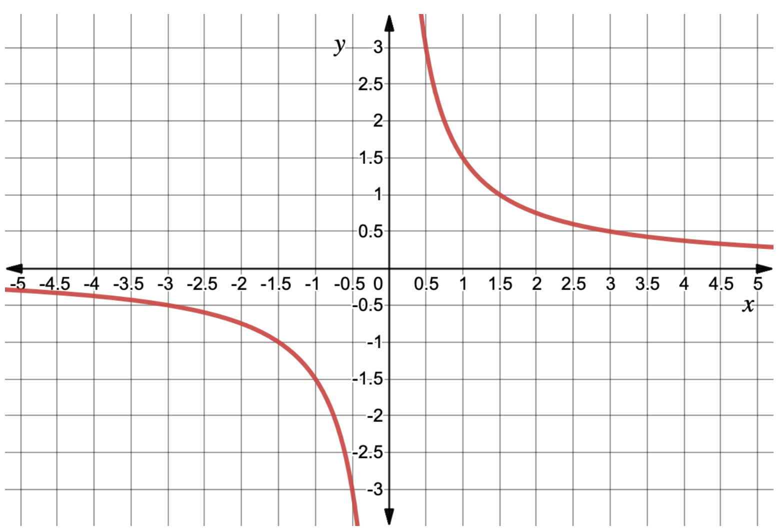
Solution: There is a vertical asymptote at $x = -1$ and a horizontal asymptote at $y = 0$. Domain: $\{x \in \mathbb{R},\, x \neq -1\}$; Range: $\{y \in \mathbb{R},\, y \neq 0\}$



https://www.desmos.com/calculator/bvbhwfifti

(b) $g(x) = \frac{3}{2x}$

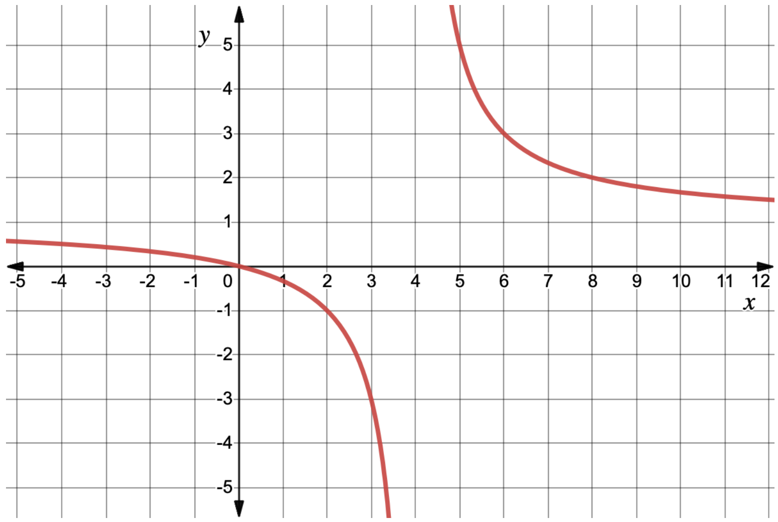
Solution: There is a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 0$. Domain: $\{x \in \mathbb{R},\, x \neq 0\}$; Range: $\{y \in \mathbb{R},\, y \neq 0\}$



https://www.desmos.com/calculator/1tov8gr6n8

(c) $h(x) = \frac{x}{x-4}$

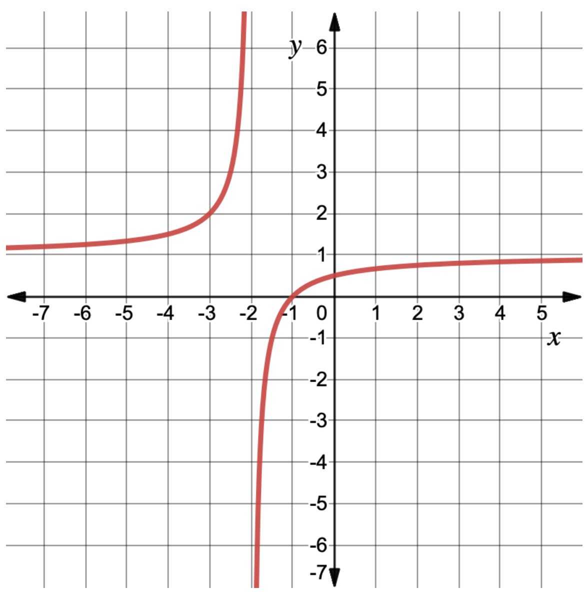
Solution: There is a vertical asymptote at $x = 4$ and a horizontal asymptote at $y = 1$. Domain: $\{x \in \mathbb{R},\ x \neq 4\}$; Range: $\{y \in \mathbb{R},\ y \neq 1\}$



https://www.desmos.com/calculator/pzniimsrs1

(d) $j(x) = \frac{x+1}{x+2}$

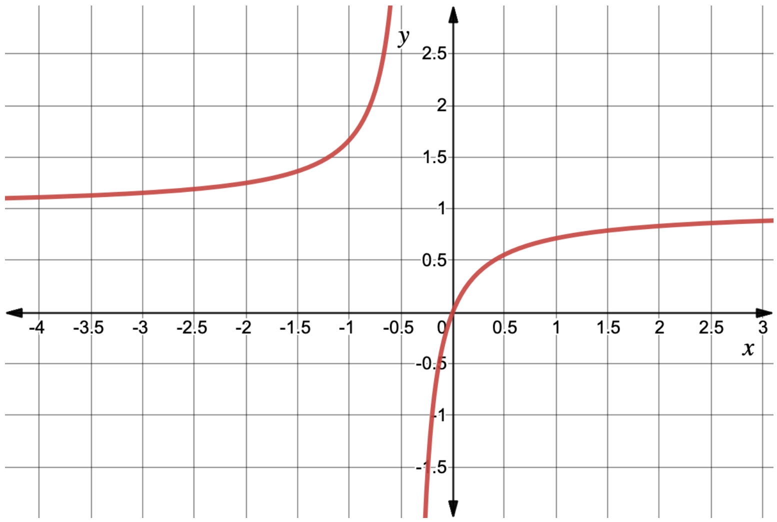
Solution: There is a vertical asymptote at $x = -2$ and a horizontal asymptote at $y = 1$. Domain: $\{x \in \mathbb{R},\ x \neq -2\}$; Range: $\{y \in \mathbb{R},\ y \neq 1\}$



https://www.desmos.com/calculator/sdqp2852iw

(e) $k(x) = \frac{5x}{5x+2}$

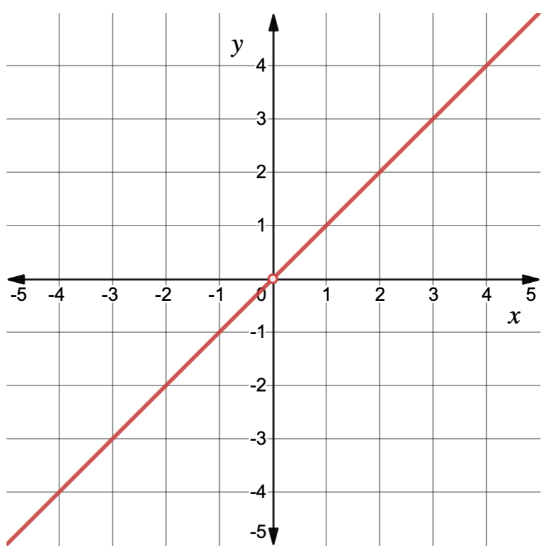
Solution: There is a vertical asymptote at $x = -\frac{2}{5} = -0.4$ and a horizontal asymptote at $y = 1$. Domain: $\{x \in \mathbb{R},\, x \neq -\frac{2}{5}\}$; Range: $\{y \in \mathbb{R},\, y \neq 1\}$.



https://www.desmos.com/calculator/qmgnrm0axu

(f) $l(x) = \frac{x^2}{x}$

Solution: There are no asymptotes, but there is a gap in the graph at the point $(0, 0)$, because $l(0)$ is undefined. Domain: $\{x \in \mathbb{R},\, x \neq 0\}$; Range: $\{y \in \mathbb{R},\, y \neq 0\}$.



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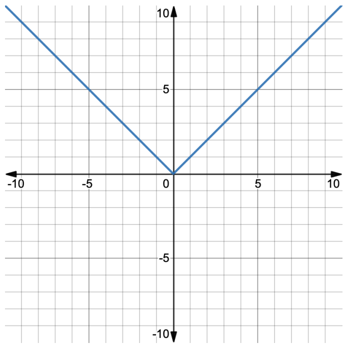
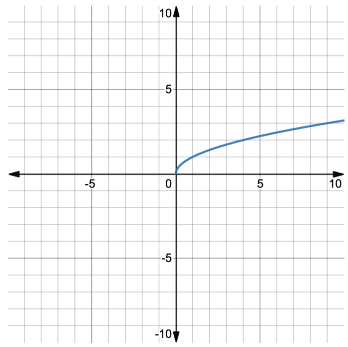
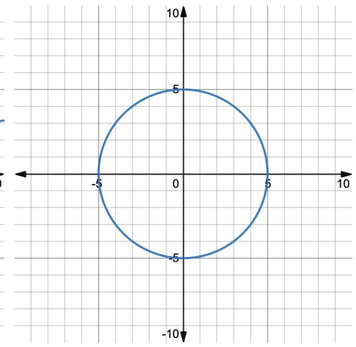
8. Using the vertical line test, identify whether each of the following graphs represents a function.



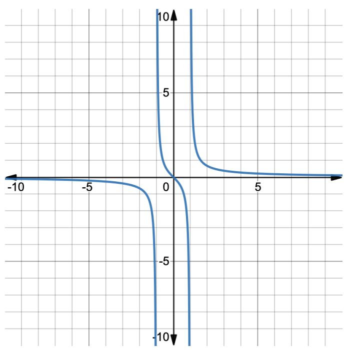
(a)



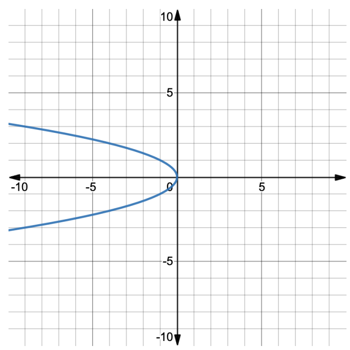
(b)



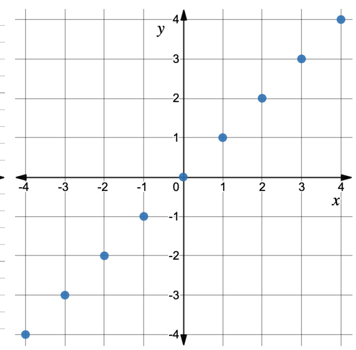
(c)



(d)



(e)



(f)

(a) yes (b) yes (c) no

(d) yes (e) no (f) yes

9. Cam is opening a side business at home baking cakes. It takes them 1.5 hours to make each cake, plus 1 hour to clean up the kitchen at the end of each day if they choose to make any cakes that day. To make sure there's still time for school, Cam can spend a maximum of 6 hours each day working on their business.

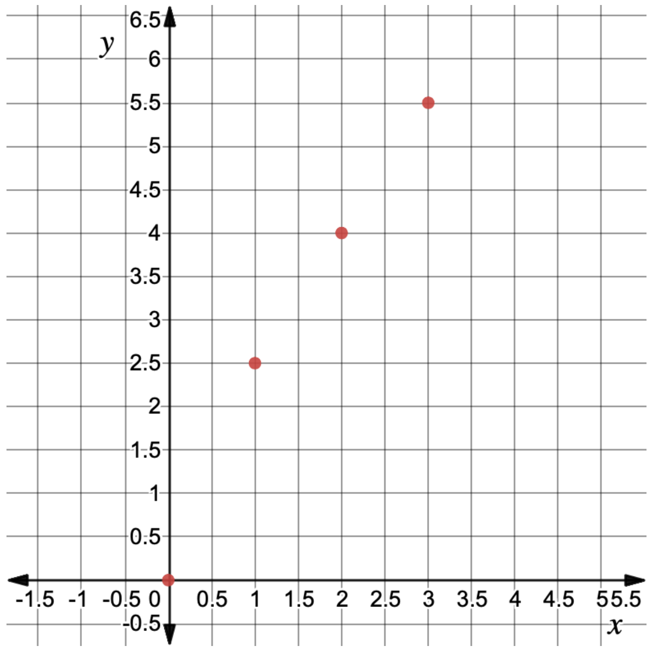
   (a) Express how long it would take Cam to make n cakes in a day as a function. Use Tas the function name (to represent "time" in hours), and use n as the variable name (to represent the "number of cakes").

   Solution:

   $$T(n) = \begin{cases} 1.5n + 1 & \text{if } n > 0 \\ 0 & \text{if } n = 0 \end{cases}$$

(b) Graph the function $T(n)$.

Solution:



(c) Using the graph, determine the maximum number of cakes that Cam can make in a day.

> *Solution:* Cam can spend a maximum of 6 hours each day working on their business, so y must be 6 or less. Also, Cam can only make whole cakes—"half" a cake would make no sense! This means that x must be a non-negative integer. Thus, the maximum number of cakes that Cam can make in a day is 3 cakes, which would take them 5.5 hours.

(d) Taking into account the context of this function, what are the domain and range of $T(n)$?

> *Solution:* $T(n)$ tells us how long it would take for Cam to make n cakes in a day. Thus, we take into account the limitations of n as noted in part (c), but nothing else—even though Cam only has time to make 3 cakes in a day, it is still true that it would take $1.5n + 1$ hours to make any other positive integer number of cakes! Therefore, we have:
> Domain: $\{n \in \mathbb{Z}, n \geq 0\}$; Range: $\{T(n) = 0, T(n) = 1.5n + 1\}$.

Hint: A function's graph doesn't always have to be a fully filled-in line! As long as it passes the vertical line test, it's a valid function.