Grade 7/8 Math Circles

February 23, 2022

Boolean Algebra - Problem Set

1. Recall that everyday logic does not translate perfectly with Boolean logic.
   (a) Respond to the following questions with an everyday response and then with a Boolean response if possible.
      i. Will you not be at the birthday party?
      ii. Would you like ice cream after lunch or after dinner?
      iii. What is the temperature outside?
   (b) Why does Boolean logic not always apply in real life?
2. Evaluate each boolean expression. Keep in mind the order of operations.
   (a) $1 \vee 0$
   (b) $0 \oplus 1$
   (c) $0 \vee 1 \wedge 0$
   (d) $\neg(1 \wedge 0)$
   (e) $\neg 0 \wedge 1$
   (f) $\neg 0 \oplus \neg(1 \wedge 0)$
   (g) $1 \wedge 0 \vee 1 \oplus 0 \vee \neg 1$
   (h) $1 \wedge (0 \vee 1) \oplus \neg(0 \vee \neg 1)$
   (i) $0 \vee 0 \vee 0 \vee 0 \vee 1 \vee 1 \vee 1$
   (j) $1 \wedge 1 \wedge 1 \wedge 0 \wedge 0$
3. Consider the following expressions.
   i. $\neg\neg B$
   ii. $B \vee B$
   iii. $B \wedge B$
   iv. $B \vee \neg B$
   v. $B \wedge \neg B$

   (a) Evaluate each of the expressions when $B = 0$.
   (b) Evaluate each of the expressions when $B = 1$.
   (c) Based on your answers to parts (a) and (b), simplify the following expressions for any Boolean value of A.
      i. $\neg\neg A$
      ii. $A \vee A$
      iii. $A \wedge A$
      iv. $A \vee \neg A$
      v. $A \wedge \neg A$

(d) Simplify the following expressions of A and B. Your findings from part (c) might be helpful.

i. $(A \vee A \vee B \vee B)$

ii. $\neg\neg((A \wedge B) \wedge (A \wedge B))$

iii. $(A \wedge \neg\neg B) \vee \neg\neg\neg(A \wedge B)$

4. We will now introduce 3 more negation operators to complete our language of boolean algebra. **NAND** ($|$), **NOR** ($\downarrow$), and **XNOR** ($\odot$).

(a) Complete the truth table provided given that

- A NAND $B = A|B = \neg(A \wedge B)$
- A NOR $B = A \downarrow B = \neg(A \vee B)$
- A XNOR $B = A \odot B = \neg(A \oplus B)$

A	B	$\neg A$	$A \wedge B$	$A\|B$	$A \vee B$	$A \downarrow B$	$A \oplus B$	$A \odot B$
0	0	1	0		0		0	
0	1	1	0		1		1	
1	0	0	0		1		1	
1	1	0	1		1		0	

(b) Rewrite the following expressions for A, B, and C using the new negation operators.

i. $A \vee \neg(B \wedge C)$

ii. $\neg(A \oplus C) \vee \neg(A \vee C)$

iii. $(\neg A \vee B) \oplus \neg(A \wedge C)$

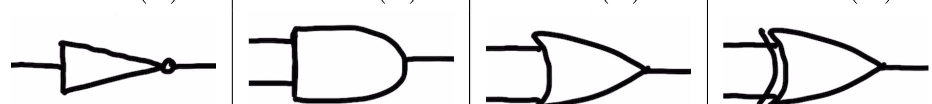
(c) Let us expand our Order of Operations to include NAND, NOR, and XNOR. (They have equal precedence as their unnegated counterparts).

Brackets $\rightarrow$ NOT $\rightarrow$ AND = NAND $\rightarrow$ XOR = XNOR $\rightarrow$ OR = NOR

Evaluate the expressions in part (b) where $A = 0, B = 1, C = 0$.

5. Determine a boolean expression that corresponds to each circuit diagram. For reference, here are the symbols again.

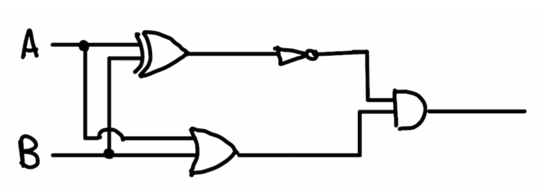
NOT ($\neg$)	**AND** ($\wedge$)	**OR** ($\vee$)	**XOR** ($\oplus$)



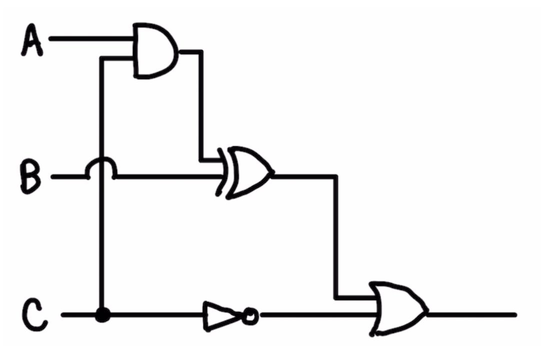
Some tips:

- Read the diagram from left to right (input to output).
- Label the output of every logic gate symbol as you go along.
- The little humps in the circuit are drawn to show one wire crossing over a different wire. They will not change the input, nor are they an operator.
- When coming up with an expression, you could use brackets at every step to avoid issues with precedence.

(a)



(b)



6. In this question you will be drawing and interpreting circuit diagrams.

   (a) Create a circuit diagram using Logic Gates for each boolean expression.

      i. $A \lor B \land C$

      ii. $A \land \neg A$

      iii. $((A \land B) \lor (B \land C)) \lor (A \land C)$

   (b) For each of the expressions/circuit diagrams, determine if there are possible input values for A, B, and C such that the output of the expression/circuit is 1.

7. Draw a circuit diagram such that the circuit output is 1 for the given inputs and required logic gates. (Hint: For some, it might be helpful to create an expression first. For others, it might be easier to go straight to the diagram.)

   (a) With inputs $A = 1$ and $B = 0$, use 1 NOT gate and 1 AND gate.

(b) With inputs $A = 0$ and $B = 0$, use 1 OR gate and 2 NOT gates.

(c) With inputs $A = 0$ and $B = 1$, use 1 OR gate and 1 AND gate.

8. The light-bulb in my kitchen responds to two light switches. Flipping either switch Up/Down will turn the light On/Off depending on if the light was Off/On before the flip.

   (a) In the context of my kitchen lighting, what could be represented with boolean variables?

   (b) When one switch is flipped Up and one switch is flipped Down, the light is On; otherwise, the light is Off. Create a boolean expression that represents the state of the light being On or Off.

   If you have a similar lighting situation at home, I recommend experimenting with all possible combinations. If not, how else could you test all possible switch-flipping combinations?

   (c) An electrical engineer insists on installing one more light switch for a total for three switches for one light-bulb. As before, flipping any of the switches in either direction will change the light from On to Off or from Off to On.

   The light On when the three switches are flipped Up. Is it possible for the light to be On when all three switches are flipped Down?

   (d) **Challenge:** A graduating class of 500 electrical engineers all have the same idea and each install another light switch for my kitchen light (503 switches in total for 1 light-bulb). As before, flipping any of the switches in either direction will change the light from On to Off or from Off to On. Assume that this is possible and that my kitchen does not catch on fire.

   The light is On when all 503 switches are flipped Up. Is it possible for the light to be On when all 503 switches are flipped down?