



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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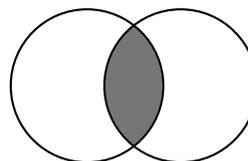
From the archives of the CEMC

October 2017

In honour of the 50th anniversary of the Faculty of Mathematics, at the beginning of each month of 2017, a set of five problems from the 54 years of CEMC contests will be posted. Solutions to the problems will be posted at the beginning of the next month. Hopefully, these problems will intrigue and inspire your mathematical mind. For more problem solving resources, please visit cemc.uwaterloo.ca.

1. *2010 Gauss Contest, Grade 8, Question 24*

Two circles each have radius 10 cm. They overlap so that each contains exactly 25% of the other's circumference, as shown. The area of the shaded region is closest to



- (A) 57.08 cm² (B) 55.24 cm² (C) 51.83 cm²
(D) 54.17 cm² (E) 53.21 cm²

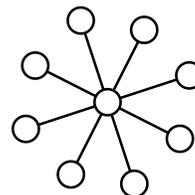
2. *1998 Fermat Contest, Question 23*

Three rugs have a combined area of 200 m². By overlapping the rugs to cover a floor area of 140 m², the area which is covered by exactly two layers of rug is 24 m². What area of floor is covered by three layers of rug?

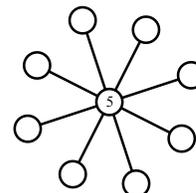
- (A) 12 m² (B) 18 m² (C) 24 m² (D) 36 m² (E) 42 m²

3. *2004 Galois Contest, Question 3*

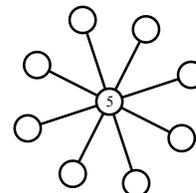
In "The Sun Game", two players take turns placing discs numbered 1 to 9 in the circles on the board. Each number may only be used once. The object of the game is to be the first to place a disc so that the sum of 3 numbers along a line through the centre circle is 15.



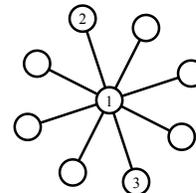
- (a) If Avril places a 5 in the centre circle and then Bob places a 3, explain how Avril can win on her next turn.



- (b) If Avril starts by placing a 5 in the centre circle, show that whatever Bob does on his first turn, Avril can always win on her next turn.



- (c) If the game is in the position shown and Bob goes next, show that however Bob plays, Avril can win this game.



4. *2011 Canadian Intermediate Mathematics Contest, Question A6*

The product of all of the positive integer divisors of 6^{16} equals 6^k for some integer k . Determine the value of k .

5. *2017 Canadian Team Mathematics Contest, Relay #3*

(a) Suppose that $x = \sqrt{20 - 17 - 2 \times 0 - 1 + 7}$. What is the value of x ?

(b) Let t be TNYWR.

If the graph of $y = 2\sqrt{2t}\sqrt{x} - 2t$ passes through the point (a, a) , what is the value of a ?

(c) Let t be TNYWR.

Suppose that

$$\frac{1}{2^{12}} + \frac{1}{2^{11}} + \frac{1}{2^{10}} + \cdots + \frac{1}{2^{t+1}} + \frac{1}{2^t} = \frac{n}{2^{12}}$$

(The sum on the left side consists of $13 - t$ terms.)

What is the value of n ?

(In a Relay, the acronym “TNYWR” in parts (b) and (c) stands for “the number you will receive”, which is the answer to the previous part. Thus, the complete information necessary to solve problems (b) and (c) depends on the answers to problems (a) and (b), respectively.)