The approach we will use to figure out the winning strategy for this game will be different than the approach used in the similar game of Pick Up Sticks with two piles. The winning strategy will be built around an understanding of what we will call winning positions.

A position of the game will be represented by an ordered triple of non-negative integers. The ordered triple \((a, b, c)\) will refer to the position where one pile has \(a\) sticks, one pile has \(b\) sticks and one pile has \(c\) sticks. We will always represent a position with an ordered triple \((a, b, c)\) satisfying \(a \leq b \leq c\). This can be done because the order of the piles does not matter. For example, if the three piles at some point have 1, 3, and 5 sticks, then the current position of the game is represented by \((1, 3, 5)\). If 3 sticks are now removed from the pile of 5, then the three piles have 1, 3, and 2 sticks but the new position is represented by \((1, 2, 3)\) rather than \((1, 3, 2)\).

A winning position is a position with the property that if we make a move to bring the game to that position, we have a winning strategy from that point forward. In other words, a winning position is a position from which our partner cannot win (unless we make a mistake later). If we move the game to a winning position, our partner cannot make a move to change it to a winning position. Furthermore, no matter what move our partner makes, there will be a move available on our next turn that changes the game back to a winning position.

By the definition of the game, we win if we pick up the last stick. Therefore \((0, 0, 0)\) is a winning position. We are going to develop a table of winning positions for our game. Since the game starts in the position \((3, 5, 7)\), we will not consider any positions \((a, b, c)\) where \(a > 3\), \(b > 5\), or \(c > 7\). Here are two observations.

**Observation 1:** From our work with Pick Up Sticks with two piles, we know that if a position has exactly two piles and these piles are equal, then it is a winning position. Also, a position having exactly two unequal piles is not a winning position. Therefore, the position \((0, k, k)\) is a winning position and the position \((0, k, \ell)\) with \(k < \ell\) is not a winning position.

**Observation 2:** Suppose we have two position \(P\) and \(Q\) and that \(P\) is a winning position. If two of the piles in \(Q\) are the same as two of the piles of \(P\) and the other pile of \(Q\) has more sticks than the other pile of \(P\), then \(Q\) is not a winning position. This is because there is a move from \(Q\) to the winning position \(P\). For example, \((3, 3, 5)\) is not a winning position because \((0, 3, 3)\) is a winning position (see previous observation), the two positions \((0, 3, 3)\) and \((3, 3, 5)\) have two pile sizes in common, and the other pile in \((3, 3, 5)\) is larger than the other pile in \((0, 3, 3)\).

This video uses these two observations to develop a table of winning positions for our game.

Here are the winning positions: \((0, 0, 0)\), \((0, k, k)\), \((1, 2, 3)\), \((1, 4, 5)\), \((2, 4, 6)\), \((2, 5, 7)\), \((3, 4, 7)\), and \((3, 5, 6)\).

Since our starting position is \((3, 5, 7)\) we can see that Player 1 has a winning strategy. Player 1 can remove one stick from any of the three piles to move the game to a winning position.

If you change the starting position of the game, you will change which player has a winning strategy. In general, if the starting position is not a winning position, then Player 1 has a winning strategy, and if the starting position is a winning position, then Player 2 has a winning strategy. The reasoning from the video can be extended to identify the winning positions in a game starting with different numbers of sticks.