Three discs, each of a different size, are arranged in a grid as shown below. Each disc starts off in its own square with the discs arranged in increasing order of size, so that the smallest disc is in the leftmost square and the largest disc is in the rightmost square.

Your goal is to reverse the order of the discs, so that the smallest disc is in the rightmost square and the largest disc is in the leftmost square, however, you must follow certain rules when moving the discs:

- At all times, each square in the grid must contain a single disc, a single stack of discs, or be empty.
- A disc may be moved on top of another disc of larger size, but not of smaller size.
- Any single disc may be moved left or right into any empty square in the grid.
- Discs can only be moved over one square at a time.
  
  For example, a disc cannot be move directly from the leftmost square to the rightmost square without passing through the middle square.

- Only one disc can be moved at a time. If there is a stack of discs in a square in the grid, then only the top disc in the stack can be moved, not the entire stack at once.

For example, here are three moves performed one after the other that follow the rules:

<table>
<thead>
<tr>
<th>Start</th>
<th>After Move 1</th>
<th>After Move 2</th>
<th>After Move 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Start Diagram" /></td>
<td><img src="image2" alt="After Move 1 Diagram" /></td>
<td><img src="image3" alt="After Move 2 Diagram" /></td>
<td><img src="image4" alt="After Move 3 Diagram" /></td>
</tr>
</tbody>
</table>

You can either move the small disc on top of the medium disc, or move the medium disc on top of the large disc.

You now cannot move the stack of two discs. You can only move the medium disc back or move the small disc over.

You cannot move the stack of two discs. You cannot move the medium disc as it would have to go on top of the small disc.

You can only move the small disc from this position.

See the next page for some problems to think about while you explore.
Problems:

1. Describe a sequence of moves that takes the three discs from the starting arrangement shown below on the left to the arrangement shown below on the right.

2. Describe a sequence of moves that takes the three discs from the starting arrangement shown below on the left to the arrangement shown below on the right, which has the discs in the reverse order.

*Solving this problem is the main goal of the activity!*

3. Now suppose you start with four discs instead of three. Just like before, they are all different sizes, and arranged in increasing order of size, with the smallest disc on the left and largest disc on the right. As in 2., you want to reverse the order of the discs, following the same rules. How can you use your solution for moving three discs from 2. to come up with a solution for moving four discs?

4. Building on the previous question, how can you use the solution for four discs to get a solution for the similar puzzle for five discs? In general, if you know how to solve the puzzle for a certain number of discs, how can you use it to solve the similar puzzle with one more disc added?

Extension: Suppose we add one more rule: stacks cannot have more than two discs in them at any time. Do you think there is still a solution to the puzzle with three discs from 2.? Do you think there is still a solution to the similar puzzle with four discs? Explain your answers.

More Info:
Check out the CEMC at Home webpage on Tuesday, May 19 for a solution to Shifting Discs.