Martin has created an irrigation system to water the fields in his farm. The water flows from a lake at the top of the hill all the way down to six fields numbered 1 to 6 at the bottom. Along the water canals, Martin has installed four water gates (A, B, C, and D), where he can direct the water to flow either to the left or to the right, but not in both directions.

An example showing how these gates can be set to have the water flow to fields 1, 2, 5, and 6 is shown to the right.

**Problem 1:** Explain how Martin can set the water gates so that water flows to fields 2, 3, and 4.

**Solution:** Notice that we do not want water to flow to field 6. That tells us gate C must be set to the left. Setting gate C to the left means it does not matter which direction we set gate D because no water will be flowing through gate D. Gate B can also be set either to the left or to the right. If gate B is set to the left, then gate A must be set to the right because we do not want water to flow to field 1. If gate B is set to the right, then it does not matter how we set gate A.

The tables below give a summary of all possible ways Martin could set the water gates so that water flows to fields 2, 3, and 4. You should check for yourself that each of these settings achieves the result.

<table>
<thead>
<tr>
<th>Gate A</th>
<th>Gate B</th>
<th>Gate C</th>
<th>Gate D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left or Right</td>
<td>Right</td>
<td>Left</td>
<td>Left or Right</td>
</tr>
</tbody>
</table>

**Problem 2:** Martin wants to set the gates so that water flows to fields 2, 3, 5, and 6.

(a) Explain why this is not possible based on how the farm is currently set up.

(b) Explain how the water canals in the farm can be adjusted in order to make this possible.

   i. Can you achieve this by removing one existing canal from the irrigation system?
   ii. Can you achieve this by adding one new canal to the irrigation system?

**Solution:**

(a) To get water to field 6, gate C must be set to the right. With gate C set to the right, we have no choice but to set gate D to the right, to ensure that water does not flow to field 4. With gates C and D set to the right, we have no choice but to set gate B to the right as well to ensure water gets to field 3. However, setting gate B to the right will also result in water flowing into field 4. Therefore, it is not possible to have water flow to only fields 2, 3, 5, and 6.

In fact, you might have noticed from the diagram that whenever water flows into field 3, it must also flow into field 4. This means it is impossible to achieve the goal in Problem 2.
(b)  

i. By removing the canal shown below, we can have water flow into fields 2, 3, 5, and 6. Removing this canal would allow water to flow into field 3 without also flowing into field 4. This was the problem we had with the original canal setup. Following the logic in the solution to part (a), we set gates B, C, and D to the right, but with this canal removed, we will now get water flowing into field 3 and *not* field 4. (It does not matter which direction we set for gate A because no water will be flowing through it.) After doing this, we will have water flowing to only fields 2, 3, 5, and 6, as desired.

![Diagram](image1.png)

ii. By adding the canal shown below, we can have water flow into fields 2, 3, 5, and 6. Adding this canal would allow water to flow into field 3 without also flowing into field 4. In this case, we should set gate A to the right, gate B to the left, and gates C and D to the right. After doing this, we will have water flowing to only fields 2, 3, 5, and 6, as desired.

![Diagram](image2.png)