Solutions to the two contest problems are provided below, including a video for the first problem.

2020 Gauss Contest, #19

Three different views of the same cube are shown. The symbol on the face opposite ⬤ is

(A) +   (B) ■   (C) ★
(D) □   (E) ○

Solution:

We begin by recognizing that there are 6 different symbols, and so each face of the cube contains a different symbol.

From left to right, let us number the views of the cube 1, 2 and 3.

Views 1 and 2 each show a face containing the symbol ★.

What symbol is on the face opposite to the face containing ★?

In view 1, □ and ○ are on faces adjacent to the face containing ★, and so neither of these can be the symbol that is on the face opposite ★.

In view 2, ■ and + are on faces adjacent to the face containing ★, and so neither of these can be the symbol that is on the face opposite ★.

There is only one symbol remaining, and so ⬤ must be the symbol that is on the face opposite ★, and vice versa.

A net of the cube is shown below.

Answer: (C)

Video

Visit the following link to view another solution to the first contest problem that uses nets: https://youtu.be/N88l8IXEiHs

See the next page for a solution to the second contest problem.
2016 Gauss Contest, #20

In the diagram, four different integers from 1 to 9 inclusive are placed in the four boxes in the top row. The integers in the left two boxes are multiplied and the integers in the right two boxes are added and these results are then divided, as shown. The final result is placed in the bottom box. Which of the following integers cannot appear in the bottom box?

(A) 16  (B) 24  (C) 7  (D) 20  (E) 9

Solution:

We begin by naming the boxes as shown to the right. Of the five answers given, the integer which cannot appear in box $M$ is 20. Why?

Since boxes $F$ and $G$ contain different integers, the maximum value that can appear in box $K$ is $8 \times 9 = 72$.

Since boxes $H$ and $J$ contain different integers, the minimum value that can appear in box $L$ is $1 + 2 = 3$.

Next, we consider the possibilities if 20 is to appear in box $M$.

If 3 appears in box $L$ (the minimum possible value for this box), then box $K$ must contain 60, since $60 \div 3 = 20$.

However, there are no two integers from 1 to 9 whose product is 60 and so there are no possible integers which could be placed in boxes $F$ and $G$ so that the product in box $K$ is 60.

If any integer greater than or equal to 4 appears in box $L$, then box $K$ must contain at least $4 \times 20 = 80$.

However, the maximum value that can appear in box $K$ is 72.

Therefore, there are no possible integers from 1 to 9 which can be placed in boxes $F, G, H$, and $J$ so that 20 appears in box $M$.

The diagrams below demonstrate how each of the other four answers can appear in box $M$.

Answer: (D)