CEMC at Home
Grade 9/10 - Monday, May 4, 2020
Contest Day 1 - Solution

Solutions to the two contest problems are provided below, including a video for the first problem.

**2020 Euclid Contest, #2(a)**

The three-digit positive integer \( m \) is odd and has three different digits. If the hundreds digit of \( m \) equals the product of the tens digit and ones (units) digit of \( m \), what is \( m \)?

*Solution:*

Suppose that \( m \) has hundreds digit \( a \), tens digit \( b \), and ones (units) digit \( c \).

From the given information, \( a, b \) and \( c \) are distinct, each of \( a, b \) and \( c \) is less than 10, \( a = bc \), and \( c \) is odd (since \( m \) is odd).

The integer \( m = 623 \) satisfies all of these conditions. Since we are told there is only one such number, then 623 is must be the only answer.

Why is this the only possible value of \( m \)?

We note that we cannot have \( b = 1 \) or \( c = 1 \), otherwise \( a = c \) or \( a = b \).

Thus, \( b \geq 2 \) and \( c \geq 2 \).

Since \( c \geq 2 \) and \( c \) is odd, then \( c \) can equal 3, 5, 7, or 9.

Since \( b \geq 2 \) and \( a = bc \), then if \( c \) equals 5, 7 or 9, \( a \) would be larger than 10, which is not possible.

Thus, \( c = 3 \).

Since \( b \geq 2 \) and \( b \neq c \), then \( b = 2 \) or \( b \geq 4 \).

If \( b \geq 4 \) and \( c = 3 \), then \( a > 10 \), which is not possible.

Therefore, we must have \( c = 3 \) and \( b = 2 \), which gives \( a = 6 \).

*Video*

Visit the following link to view a discussion of a solution to the first contest problem: https://youtu.be/dJ6d0ILAGwE

**2020 Canadian Team Mathematics Contest, Team Problem #6**

On Fridays, the price of a ticket to a museum is $9. On one particular Saturday, there were 200 visitors to the museum, which was twice as many visitors as there were the day before. The total money collected from ticket sales on that particular Saturday was \( \frac{4}{3} \) as much as the day before. The price of tickets on Saturdays is \( k \). Determine the value of \( k \).

*Solution:*

There were 200 visitors on Saturday, so there were 100 visitors the day before. Since tickets cost $9 on Fridays, the total money collected on Friday was $900.

Therefore, the amount of money collected from ticket sales on the Saturday was \( \frac{4}{3} \times 900 = 1200 \).

Since there were 200 visitors on Saturday, the price of tickets on that particular Saturday was \( \frac{1200}{200} = 6 \). Therefore, the value of \( k \) is 6.

*It turns out that you do not need to know the number of visitors to the museum on the Saturday to solve the problem. If this number (200) changes, but all other conditions in the problem are kept the same, then the answer will still be \( k = 6 \). Can you see why?*