Shady Circles - Solution

Problem 1 Solution

We will find the area of the shaded region by subtracting the area of the smaller circle from the area of the larger circle.

Let $R$ be the radius of the larger circle and $r$ be the radius of the smaller circle.

Since the diameter of the smaller circle is the radius of the larger circle, we have $R = \frac{20}{2} = 10$ and $r = \frac{10}{2} = 5$.

$A_{\text{larger}} = \pi R^2 = \pi (10)^2 = 100\pi$

$A_{\text{smaller}} = \pi r^2 = \pi (5)^2 = 25\pi$

Therefore, the area of the shaded region is $100\pi - 25\pi = 75\pi$.

Problem 2 Solution

To find the area of the shaded region, we will find the area of the sector of the circle with arc $AB$ and subtract the area of $\triangle AOB$. Note that the triangle is a right isosceles triangle and therefore, the base and height are both equal to the radius which is 2.

$A_{\text{whole circle}} = \pi r^2 = \pi (2)^2 = 4\pi$

$A_{\text{sector}} = \left(\frac{90}{360}\right) 4\pi = \pi$

$A_{\text{triangle}} = \frac{bh}{2} = \frac{(2)(2)}{2} = 2$

Therefore, the area of the shaded region is $\pi - 2$.

Problem 3 Solution

There are a few different ways to approach this problem. We will outline two approaches. Each of these approaches relies upon the following facts that we will not prove:

1) The two diagonals of the inscribed square intersect at the centre, $O$, of the circle.

2) The two diagonals of the inscribed square bisect each other and meet at right angles.
**Approach 1:** Recognize that the shaded region in this problem consists of four identical shaded regions, each having an area that can be calculated by subtracting the area of a triangle from the area of a sector of a the circle (as in Problem 2).

The final calculation is as follows: \[ \text{Area} = 4 \left( \frac{\pi (5)^2}{4} - \frac{5^2}{2} \right) = 25\pi - 50. \]

**Approach 2:** Recognize that the area of the shaded region is the area of the circle minus the area of the inscribed square.

The area of the circle \( \pi r^2 = \pi (5)^2 = 25\pi. \)

Let \( s \) be the side length of the square as shown in the figures below.

Note that \( \triangle BOC \) is a right isosceles triangle. Therefore, its base and height are both equal to the radius which is \( r = 5 \). We can calculate the value of \( s^2 \) as follows:

Using the Pythagorean Theorem on \( \triangle BOC \), we get

\[
\begin{align*}
 s^2 &= r^2 + r^2 \\
 s^2 &= (5)^2 + (5)^2 \\
 s^2 &= 25 + 25 \\
 s^2 &= 50
\end{align*}
\]

Alternatively, using the Pythagorean Theorem on \( \triangle BAD \), with diameter \( d = 10 \), we get

\[
\begin{align*}
 d^2 &= s^2 + s^2 \\
 10^2 &= 2s^2 \\
 100 &= 2s^2 \\
 50 &= s^2
\end{align*}
\]

Each of these calculations tells us that the area of the square is \( s^2 = 50 \).

Therefore, the area of the shaded region is \( 25\pi - 50 \).

**Problem 4 Discussion**

One way to find the area of the shaded region is to observe that it is made up of two identical regions as shown below. You can find the area of each of the regions using a similar method to that in the solution to Problem 2, although the area of the triangle will not be as easy to calculate in this case.

We leave the details to you, but give the key values in the calculations here.

Area of triangle \( AOB \) is \( \frac{1}{2}(10)(\sqrt{75}) \)

Area of sector \( AOB \) is \( \frac{60}{360}(\pi(10)^2) \)