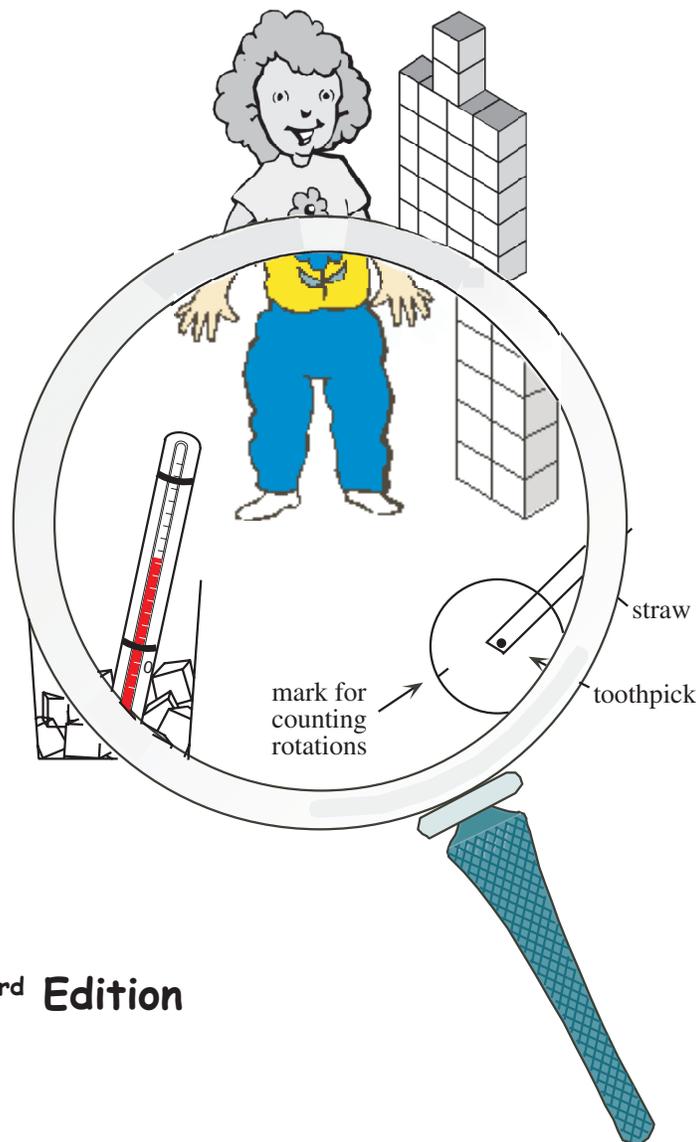


# *Invitations to Mathematics*

## *Investigations in Measurement*

### *“Measures and Magnitudes”*

Suggested for students  
at the  
Grade 6 level



3<sup>rd</sup> Edition



An activity of  
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## Preface

The Centre for Education in Mathematics and Computing at the University of Waterloo is dedicated to the development of materials and workshops that promote effective learning and teaching of mathematics. This unit is part of a project designed to assist teachers of Grades 4, 5, and 6 in stimulating interest, competence, and pleasure in mathematics among their students. While the activities are appropriate for either individual or group work, the latter is a particular focus of this effort. Students will be engaged in collaborative activities which will allow them to construct their own meanings and understanding. This emphasis, plus the extensions and related activities included with individual activities/projects, provide ample scope for all students' interests and ability levels. Related "Family Activities" can be used to involve the students' parents/care givers.

Each unit consists of a sequence of activities intended to occupy about one week of daily classes; however, teachers may choose to take extra time to explore the activities and extensions in more depth. The units have been designed for specific grades, but need not be so restricted. Activities are related to the Ontario Curriculum but are easily adaptable to other locales.

"Investigations in Measurement" is comprised of activities which explore estimation and measurement, and the selection of appropriate tools and units. Measurement provides the means to describe and analyse the everyday world in concrete terms, from grocery shopping through car assembly to building a space module. The activities involve making and testing hypotheses, and other forms of problem solving, as well as connecting strands of mathematics to each other and to other curriculum areas.

## Acknowledgements

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## Table of Contents

<b>Preface</b> .....	i
<b>Acknowledgements</b> .....	ii
<b>Table of Contents</b> .....	iii
<b>Overview</b> .....	1
COMMON BELIEFS.....	1
ESSENTIAL CONTENT .....	1
CURRICULUM EXPECTATIONS.....	2
PREREQUISITES .....	3
LOGOS .....	3
"SNIPPETS" AND RULE OF THUMB DESCRIPTION.....	3
MATERIALS.....	4
LETTER TO PARENTS .....	5
<b>Activity 1: Shapes of People</b> .....	7
<b>Activity 2: Holes for Everything</b> .....	10
<b>Activity 3: Inside and Outside Boxes</b> .....	13
<b>Activity 4: Hot and Cold</b> .....	17
<b>Activity 5: A Mini-Olympics</b> .....	21
<b>BLM 1: What Shape Are You In?</b> .....	24
<b>BLM 2: Getting in Shape</b> .....	25
<b>BLM 3: A Decimetre Ruler</b> .....	26
<b>BLM 4: Triangle Dot Paper</b> .....	27
<b>BLM 5: Thinking About Holes</b> .....	28
<b>BLM 6: Bigger Than You Think</b> .....	29
<b>BLM 7: A Template</b> .....	30
<b>BLM 8: Centimetre Grid</b> .....	31
<b>BLM 9: Making Cardboard Boxes</b> .....	32
<b>BLM 10: Cylinders Every Which Way</b> .....	33
<b>BLM 11: Making a Measuring Bottle</b> .....	34
<b>BLM 12: Ice Buckets</b> .....	35
<b>BLM 13: Mini-Olympics, Events 1, 2, and 3</b> .....	36
<b>BLM 14: Mini-Olympics, Events 4, 5, and 6</b> .....	37
<b>BLM 15: Mini-Olympics, Events 7 and 8</b> .....	38
<b>BLM 16: Mini-Olympics Score Cards</b> .....	39
<b>BLM 17: Decimetre Trundle Wheel</b> .....	40
<b>BLM 18: Trundle Wheel Templates</b> .....	41
<b>Solutions &amp; Notes</b> .....	42
<b>Suggested Assessment Strategies</b> .....	47
<b>Other Resources</b> .....	53



## Overview



### COMMON BELIEFS

These activities have been developed within the context of certain beliefs and values about mathematics generally, and measurement specifically. Some of these beliefs are described below.

Measurement provides the means for describing and quantifying our everyday lives. As such, it is rich with opportunities for exploring both mathematical concepts and their applications in the ‘real’ world.

Dynamic interaction between students and their environment is essential to developing skill with the processes of measurement and deriving the mathematical possibilities. Concrete, hands-on activities involving estimating, then measuring, in both non-standard and standard units, promote awareness of what units and tools are appropriate for a specific task, why standard units are necessary, and that all measurements are approximations with varying degrees of human and instrument error. Activities with ordinary objects/attributes (e.g. eraser, desk, blackboard, height, heartbeat, gasoline consumption, shadows, packaging, etc.) will provide limitless opportunities for problem-solving, involving concepts such as length, area, volume, time rates, ratio and proportion, similarity and congruence, as well as connecting mathematics directly to the physical world.

Justifying their own reasoning and discovering the patterns and logical connections which lead to mathematical formulas, or to a deeper understanding of their everyday world, increases the students’ ability to reason analytically and to express their thoughts clearly and concisely.

### ESSENTIAL CONTENT

In the activities herein, students will explore the process of measurement and its implications in a variety of contexts, both inside and outside the classroom, with the goal of developing a solid foundation for using instruments and formulas with skill and precision, and analysing the meaning of their measurements. In addition, there are Extensions in Mathematics, Cross-Curricular Activities and Family Activities. These may be used prior to or during the activity as well as following the activity. They are intended to suggest topics for extending the activity, assisting integration with other subjects, and involving the family in the learning process.

During this unit, the student will:

- explore the volume and surface area of the human body;
- estimate the cross-sectional area of three-dimensional objects;
- use careful measurement to cut a hole in a piece of paper large enough for a bicycle to pass through;
- construct a variety of rectangular prisms and compare their volumes and surface areas;
- observe, record, and display (graphically) data on how temperature changes in a variety of situations;
- participate in a measurement mini-olympics involving estimating and measuring lengths, areas, volumes, angles, and time intervals;
- use mathematical language to express their results;
- work together to achieve success.



## CURRICULUM EXPECTATIONS

ACTIVITY	DESCRIPTION OF THE ACTIVITY	CURRICULUM EXPECTATIONS
<b>Activity 1</b>  <b>Shapes of People</b>	<ul style="list-style-type: none"> <li>exploring and comparing unusual measurements such as volume and skin surface area</li> <li>using the decimetre as a unit of length</li> </ul>	<ul style="list-style-type: none"> <li>identify relationships between and among measurement concepts (linear, square, cubic)</li> <li>select among commonly used SI units in solving problems</li> <li>make simple conversions between metric units</li> </ul>
<b>Activity 2</b>  <b>Holes for Everything</b>	<ul style="list-style-type: none"> <li>estimating the size of hole needed to allow various objects to pass through</li> <li>exploring a unique way to cut a surprisingly large hole in a standard piece of copy paper</li> </ul>	<ul style="list-style-type: none"> <li>identify relationships between and among measurement concepts (linear and square)</li> <li>understand the relationship between area and lengths of sides for a rectangle</li> <li>solve problems related to calculation and comparison of perimeter and area</li> </ul>
<b>Activity 3</b>  <b>Inside and Outside Boxes</b>	<ul style="list-style-type: none"> <li>constructing rectangular prisms from centimetre grid paper, comparing their sizes by calculating their volumes and surface areas</li> <li>exploring attributes of boxes used for various commodities (e.g., size versus weight of contents)</li> </ul>	<ul style="list-style-type: none"> <li>estimate, measure, and record ... the volume of prisms</li> <li>estimate and calculate the volume of rectangular prisms</li> <li>select among commonly used SI units ... in solving problems</li> </ul>
<b>Activity 4</b>  <b>Hot Water Bottles</b>	<ul style="list-style-type: none"> <li>observing temperature changes over time in various situations (e.g., hot water in insulated bottles, melting ice, under paper in a sunny window)</li> <li>constructing graphs to display the data collected and help analyse temperature behaviour</li> </ul>	<ul style="list-style-type: none"> <li>organize and record data on tally charts*</li> <li>make inferences based on analysis of graphs*</li> </ul> <p>[See also the note at the start of Activity 4 re: Science connections]</p>
<b>Activity 5</b>  <b>A Mini-Olympics</b>	<ul style="list-style-type: none"> <li>estimating and measuring lengths, areas, volumes, angles and time intervals in a variety of activities (e.g., fist volume, footprint area, finger-thumb angles)</li> </ul>	<ul style="list-style-type: none"> <li>estimate, measure, and record [lengths, volumes and times] in appropriate metric units</li> <li>relate time and distance and speed</li> </ul>

\* These expectations come from curriculum expectations for Data Management

*“Curriculum Expectations” are based on current Ontario curricula.*

## Overview

### PREREQUISITES

Students should be familiar with metric units and measurement terms (e.g., area, perimeter, volume, cm, m, L, kg, degrees) as well as the use of various measuring devices such as a thermometer, a clock, a protractor and a ruler. Beyond this basic knowledge there are no prerequisites for the activities in this booklet.

### LOGOS

The following logos, which are located in the margins, identify segments related to, respectively:

#### Problem Solving



#### Communication



#### Assessment



#### Use of Technology



### SNIPPETS

“Snippets” that appear as small notebook pages in the margins are bits of data somehow related to the measurement tasks the students are being given. Sometimes these snippets will include a problem posed for the students. For others, questions will no doubt come to the teacher’s mind even as he/she is sharing the snippet with students. Students themselves may identify related questions that they would be interested in pursuing. It is hoped that students will find these bits of information interesting and will realize how frequently measurements are used in everyday life.

### RULES OF THUMB

#### R of T

“Rules of Thumb” are ways to help in estimating. For example, the rule-of-thumb “Two pages written by hand will give one page when typewritten” will give an author some idea of the length of his/her erudite article, so he/she knows when the article has reached a permissible length. Rules of Thumb (R of T) have been placed in margins (on file cards) alongside the Activity notes. They can be used as jumping off points for good problems, or just enjoyed for their (possible) values. A worthwhile activity is trying to decide whether each R of T is valid. All R of T’s in this book are gleaned from “Rules of Thumb” and “Rules or Thumb -2” by Tom Parker. See “Other Resources” on page 47 for more detail.



**MATERIALS**

<b>ACTIVITY</b>	<b>MATERIALS</b>
<b>Activity 1</b> <b>Shapes of People</b>	<ul style="list-style-type: none"> <li>• Copies of BLMs 1, 2, 3</li> <li>• Tape, scissors</li> <li>• Copies of BLM 4 (optional)</li> <li>• Rulers or metre tapes</li> <li>• Linking cubes/centicubes/wooden blocks/sugar cubes (optional)</li> <li>• A model of 'Charlie Cube' from BLM 1 (optional)</li> </ul>
<b>Activity 2</b> <b>Holes for Everything</b>	<ul style="list-style-type: none"> <li>• Copies of BLMs 5, 6 for each pair/group</li> <li>• Copy paper (8" x 11"), rulers, scissors, tape</li> <li>• Copies of BLMs 7, 8 (optional)</li> </ul>
<b>Activity 3</b> <b>Inside and Outside Boxes</b>	<ul style="list-style-type: none"> <li>• Collection of boxes from home (e.g., cereal, rice, crackers, cake mixes, tea bags) that indicate the mass of the contents</li> <li>• Copies of BLMs 8 and 9 for each pair/group</li> <li>• Scissors, tape</li> <li>• Centimetre cubes (constructed as on BLM 9)</li> <li>• Copies of BLMs 10 and 11 (optional)</li> <li>• Pouring material (e.g., water, rice, sand)</li> </ul>
<b>Activity 4</b> <b>Hot and Cold</b>	<ul style="list-style-type: none"> <li>• 4-6 identical bottles, fabric pieces to wrap around them, elastics to hold them in place, and a Celcius thermometer</li> <li>• Thermometer in °C for each group</li> <li>• Copies of BLM 12 (one per group for each experiment)</li> <li>• Container of ice for each group</li> <li>• Graph paper</li> </ul>
<b>Activity 5</b> <b>A Mini-Olympics</b>	<ul style="list-style-type: none"> <li>• Copies of BLMs 13, 14, 15, 16, 17 for each team</li> <li>• Copies of BLM 18 (optional)</li> <li>• Materials for whichever events are to be used</li> </ul>



## LETTER TO PARENTS

### SCHOOL LETTERHEAD

#### DATE

Dear Parent(s)/Guardian(s):

For the next week or so, students in our classroom will be participating in a unit titled “Measures and Magnitudes”. The classroom activities will focus on expanding students’ understanding of measurement and estimation while constructing models of themselves based on their own measurements, and exploring containers of various types and sizes. The emphasis will be on developing a greater understanding of the nature of measurement and the validity of estimating.

You can assist your child in understanding the relevant concepts and acquiring good measurement skills by working together to perform simple tasks (e.g., cooking from a recipe, sewing a tablecloth, building a chest, ...), helping to explore everyday ways measurement is used.

Various family activities have been planned for use throughout this unit. Helping your child with the completion of these will enhance his/her understanding of the concepts involved.

If you work with measurement in your daily work or hobbies, please encourage your child to learn about this so that he/she can describe these activities to his/her classmates. If you would be willing to visit our classroom and share your experience with the class, please contact me.

Sincerely,

Teacher's Signature

#### **A Note to the Teacher:**

*If you make use of the suggested Family Activities, it is important to schedule class time for sharing and discussion of results.*



## Activity 1: Shapes of People

### Focus of Activity:

- Estimating and measuring with decimetres and cubic centimetres/decimetres
- Estimating body size

### What to Assess:

- Accuracy of measurements
- Reasonableness of estimates
- Facility with decimetre as a unit

### Preparation:

- Make copies of BLMs 1, 2, and 3 for each student.
- Provide tape and scissors.
- Make copies of BLM 4 (optional).
- Provide rulers or metre tapes.
- Provide linking cubes/centicubes/wooden blocks/sugar cubes (optional).
- Prepare a model of “Charlie Cube” as on BLM 1 (optional).

### Activity:

*NOTE: You will need a collection of materials for Activity 3, Extension 1. Materials include a variety of boxes, such as cereal, rice, tea bags, instant pudding, crackers, and cake mixes that have the mass of the contents listed on the box.*

Since Grade 6 students have had some experience with metric measurement, and have, no doubt, measured their heights, weights, and hand spans before, this Activity explores other “personal” measures such as body volume and skin surface.

*Ratio is included in the problems on BLM 2 as a way of comparing body measurements. If students have not encountered the word or the concept before you may wish to introduce ratio as a form of comparison before assigning BLM 2.*

Distribute copies of BLM 1 and read over #1 with students. Most adults would be rectangles according to this definition, but this may not be true for Grade 6 students. There are just two measurements made for each student, so this part of the Activity should not take long. Students should work in pairs or small groups to measure each other, though each student should complete his/her own copy of BLM 1. As students finish #1, have them indicate on a class chart whether they are rectangles or squares. When the chart is complete, students will be able to answer 1(c).

If you have built a model of “Charlie Cube” (out of linking cubes or sugar cubes or wooden blocks or any available cubes), students can refer to it as they are counting cubes for #2. Some students may have difficulty interpreting the diagram and counting cubes from it. For example, they may count the top head cube as 3 cubes because 3 surfaces are showing. Since Charlie Cube is only one cube thick, have students count the number of cubes on Charlie’s front surface to determine the total number of cubes used. Top and side surfaces are shaded to make this easier.



*Comments in italics are explanatory, and need not be conveyed to the students.*

*See page 3 for a description of “Snippets”, an example of which is given here.*

The index finger of the Statue of Liberty is 24 dm long. Compare this with the length of your index finger.

### Problem Solving





## Activity 1: Shapes of People

Throughout this activity the cubic figures are one cube thick, thus ignoring the thickness of the human figure. This will mean that students' estimates of their body volume and skin area will be on the low side. If students raise this issue, have them consider the problems of trying to design a "Cubic Me" of different thicknesses. For example, one's head is about 2 dm thick, but the neck is only about 1 dm thick, as are the arms, while the torso may be close to 3 dm thick.

### Problem Solving



If students have difficulty with #2(a), tell them that one of the measurements they made for #1 will help them. Since they measured their heights in #1 they should be able to calculate the size of each block if they were 12 cubes tall, like Charlie.

Students may find the size of cubes computed for (b) and (c) will be different but may need some assistance in explaining why, to answer part (d). This will happen because students have proportions different from Charlie's. The ratio of Charlie's head and neck to his arm length is 2(cubes) to 5(cubes). Students may discover that their own similar measurements are closer to 20 (cm) to 70 (cm) or 2 to 7. From these measurements, they would calculate each "head cube" as being 10 cm tall, and each "arm cube" as 14 cm tall.

When BLM 1 has been completed, and discussed, distribute BLM 2 and 3. The measurements on BLM 2 are to be given in decimetres. This is a unit that students may not be familiar with. However, it is a useful unit because it is larger than 1 cm, yet not as unmanageable for short lengths as 1 m. An adult hand is approximately 10 cm or 1 dm across the hand at the widest part. Some students may find this is true for their hands as well. Knowing this gives them a personal referent for estimating short lengths when they have no ruler available.

BLM 3 provides several decimetres for students to cut out and fasten together. If students will be taping the decimetres together they should cut off the tabs and tape ends of the pieces together. The tabs are provided for overlap in case students will be gluing the decimetres together. Have students fasten decimetres of different shading together. This makes each decimetre highly visible as a unit and will give students a useful visual image of a decimetre throughout this Activity. With all the decimetre strips together, students will have a measuring tape that is 12 dm or 1.2 m long.

*Another value of the decimetre is that  $1 \text{ dm}^3 = 1 \text{ L}$ , and 1 L of water weighs 1 kg.*

The measurements asked for in #1 and #2 on BLM 2 are measurements that were used to build Charlie Cube and will be used by the students to build a "Cubic Me". When students compare their measurements with those of Charlie Cube, they will probably find several differences. For example, Charlie Cube's head is just 1 dm from ear to ear and students should find that theirs are closer to 2 dm.

The albatross has a wing-spread up to 3.3 m (about the size of a small car).

The lowest branch of a Giant Sequoia is the size of another tree, about 21 m tall, and 2 m in diameter.

## Activity 1: Shapes of People

Students can compare the proportions of the child shown next to Charlie Cube on BLM 2. Students who have younger siblings could measure them (as for #1 on BLM 2) and compare the ratios of these measurements with their own and with Charlie Cube's. They should realize that, as children grow, their proportions change.

The ratios of Charlie's measurements should be easy to read from the diagram on BLM 1 since we are assuming that each cube is 1 dm. Students are then asked to use the ratios of their own measurements to design a cubic figure closer to their own measurements. If cubes are available for building these figures, students should be encouraged to build them. Otherwise a sketch will suffice. Students will find different ways to draw their own "Cubic Me".

If students have used isometric (equilateral triangle) dot paper in the past to draw three-dimensional figures, they should be encouraged to draw their "Cubic Me" on this. (BLM 4). See 'Solutions and Notes' for possible diagrams.

A discussion of the Challenge (#5 (a) and (b) on BLM 2) can explore such issues as how the volume and skin area were estimated, whether or not the answers were a surprise, and whether or not the estimates are reasonable.

### Cross-curricular Activities:

1. Have students research other body measures such as length of blood vessels, length of intestines, and volume of blood. For example, an adult has approximately 100 000 km of blood vessels; main arteries are at least 30 mm in diameter; coronary arteries are about the size of a drinking straw; and the average capillary is 0.01 mm in diameter.

### Family Activities:

1. Students could measure family members to determine if they are squares or rectangles. Ask students if the shape of a person varies with age, and, if so, how.
2. If students have pets, they could design cubic model of those pets, and then describe to the class what measurements they needed and how they took those measurements. For students who do not have pets, suggest they model a stuffed animal or another family member.

### Other Resources:

For additional ideas, see annotated Other Resources list on page 54, numbered as below.

12. Honoring Traditions: Making Connections with Mathematics through Culture by Deborah McLean.



### Problem Solving



The average adult has about  $1.6 \text{ m}^2$  or  $160 \text{ dm}^2$  of skin.

The hummingbird can flap its wings 75 times per second. The honeybee can do this 250 times per second. How about you?



## Activity 2: Holes for Everything

### Focus of Activity:

- Creating large perimeters from small areas

### What to Assess:

- Correct and accurate use of ruler to measure and draw lines
- Use of mathematical language

### Preparation:

- Make copies of BLMs 5 and 6 for each pair/group.
- Provide copy paper (8" x 11"), rulers, scissors, tape.
- Make copies of BLMs 7 and 8 (optional).

### Activity:

#### INTRODUCTION:

Distribute copies of BLM 5, and sheets of copy paper 8" x 11" (21.5 cm x 28 cm). Students should also have rulers. Have students measure the copy paper to familiarize (refamiliarize?) themselves with the size of one centimetre.

Read problem 1 on BLM 5 with them and give each group or pair an opportunity to decide on an answer. Ask students how they arrived at their answers. For example,

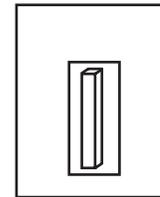
Did they estimate?

Did they measure?

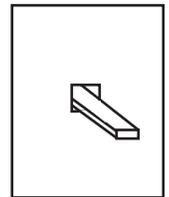
Did they subtract 1 cm (for the frame) from each side of the paper?

Note whether students describe the largest hole using linear units (e.g., 26 cm by 19.5 cm) or using area units (e.g., 500 cm<sup>2</sup>).

Tell them that for the next two problems (2 and 3) they may measure anything they need to, but they are not to cut the paper. Students may ask about the orientation of the objects. For example, does the "ruler hole" need to be big enough for the ruler to go through lying flat or on end. Similarly, they may ask if "math book" means text book or work book. Problem 4 may raise the question, "Which edges are taped together?" Tell the students that they are to make these decisions



Like this?



Like this?

or

for themselves. When they explain their answers they should indicate these decisions and how they affected their answers. Allow time for this work to be completed. Discuss answers and reasons with the students. See "Solutions and Notes" for more on this.

Distribute copies of BLM 6 and sheets of copy paper. Students should first cut off the strip of paper as shown in Figure 1. This strip is removed only so that the other measurements can be whole numbers of centimetres. The overall result will not be changed substantially if this strip is not removed. Once this strip is removed, students should do all the other measuring and drawing before doing any more cutting.

### Problem Solving



### Communication



## Activity 2: Holes for Everything

Diagrams 4 and 6 do not show all the lines to be drawn. They simply show the orientation. Students should draw, respectively, 10 and 9 lines as given in instructions (d) and (f). Students should be cautioned to be very careful as they unfold the paper. It will tear easily. Tape may be needed for repairs.

*If students have trouble measuring and drawing the lines, you may wish to give them copies of the template on BLM 7. Have them cut off the strip as shown in Figure 1 on BLM 6. Then have them fold their given sheet of copy paper in half, cut out the template, and paste it onto the copy paper, putting the indicated line along the fold. Tell them to cut on the heavy lines. Alternatively, give them copies of BLM 8, (centimetre grid paper) which can be used as a guide for drawing the lines.*

Discuss with students any changed answers to problem 2 on BLM 5 using this new “hole” in the paper. Ask if more than one student could fit through the hole. (This is #2(a) on BLM 6.) Have them predict the number of students who would fit in the hole and give their reasons. Then allow them to test their theories. This is a good outdoor or gymnasium activity.

Spreading out the “frame” of the hole to count floor tiles provides an opportunity to talk about the concept of area. Extension 2 below takes this idea further. Students should find that a square will give the greatest area.

### Extensions in Mathematics:

1. Ask students how they could make the hole in the paper even bigger. Possible responses :  
Cut the lines closer together.  
Mark the lines in parts (c) and (e) closer to the edges.  
Ask if they think it would make any difference if the paper were folded in half the other way to start. Have students test this and explain their results.
2. Have students spread the frames out to make rectangles and record the length and width of each rectangle they make.

*Because of the nature of the frame, the “rectangles” will probably not have straight sides. However, since this is an exercise in approximation, this will not be critical. Have students give measurements as nearly accurate as they can.*

To have a measure of area they could count the number of floor tiles in the frame for each rectangle. Have them determine which rectangle has the greatest area.

### Family Activities:

1. Ask students to try this “hole cutting” as on BLM 6 at home using a page of a newspaper to start. Have them predict how big a hole they can cut. Have them identify some things at home that will just barely fit in the frame.



### Problem Solving



The Brown Bear of northern Russia is almost 3 m tall. Could one of them fit through a hole in your paper?

### Problem Solving



One Golden Eagle nest, after 35 years of use, was 25 dm across and 36 dm deep. Could such a nest fit into a hole in your paper?

**Activity 2: Holes for Everything****Other Resources:**

For additional ideas, see annotated Other Resources list on page 54, numbered as below.

6. Mousemaze Tournament: Connecting Geometry and Measurement by Shirley Curtis.
8. How Big Was the Cat?, by L.E. Sakshaug and K.A. Wohlhuter.



### Activity 3: Inside and Outside Boxes

#### Focus of Activity:

- Comparing box dimensions with volume and surface area

#### What to Assess:

- Accurate construction of boxes
- Accurate calculation of volumes and surface areas
- Reasonableness of estimates and reasons for these estimates

#### Preparation:

- Make a collection of boxes of different kinds brought from home (e.g., cereal, rice, crackers, cake mixes, tea bags, instant pudding) that indicate the mass of the contents.
- Make copies of BLM 8 (centimetre grid) and BLM 9 for each pair/group.
- Provide scissors, tape.
- Make up 2 or 3 centimetre cubes as shown on BLM 9, #1.
- Make copies of BLMs 10, and 11 (optional).
- Provide pouring material (e.g., water, rice, sand) for use with BLMs 10 and 11 (optional).

#### Activity:

Review the concept of volume with students by displaying two or three boxes from the classroom collection and asking such questions as

- Which of these do you think will hold more/the most?
- What units do we use to describe how much a container will hold? (Students may reply “cubic centimetres” or “litres” among other possibilities; since the activity focusses on cubic centimetres, you may wish to mention this)
- What word(s) do we use to talk about how much something holds or how much space it takes up? (Students may answer “volume” or “capacity” or even “weight”.)

The activity will focus on volume, but Extension 1 deals with weight/mass.

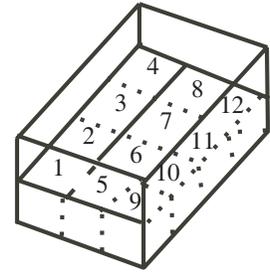
*Note that although “weight” and “mass” are two different measures, for Grade 6 students this often leads to confusion since in everyday English people use the word “weight” when they really should use “mass”. We say someone weighs 50 kg when we really mean he/she has a mass of 50 kg; we buy meat in kilograms and call it the weight. On the moon the person’s mass is the same as on earth but the weight is less since gravitational pull is less. In this unit we will use the term “weight” to make the measure easily understood by the students.*

Display the centimetre cubes you have made. Ask students what each cube should be called if it is 1 cm by 1 cm by 1 cm. Tell them they will be constructing some boxes and calculating their volumes in cubic centimetres. They will also be constructing some centimetre cubes to help them determine the volume. You may wish to review a technique for using the small cubes to determine volume (See box below) or let students decide for themselves how they can do this.

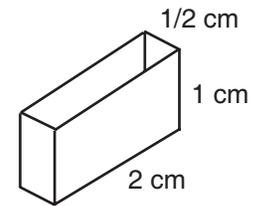


### Activity 3: Inside and Outside Boxes

One way to determine volume using centimetre cubes is to determine how many it would take to cover the bottom of the box with a layer of such cubes. The box at right needs 12 cubes for one layer. Then determine how many such layers will be needed to fill the box. It can be seen that 2 layers are necessary for this box. Thus the volume is 24 .



*Note that “cubic centimetre” and “centimetre cube” are different. A “centimetre cube” is a cube measuring 1 cm by 1 cm by 1 cm. Such a cube has a volume of one “cubic centimetre”, but boxes other than cubes can have a volume of one “cubic centimetre”. In other words, a “cubic centimetre” does not have to be a cube. For example, a box 1 cm by 1 cm by 2 cm has a volume of one cubic centimetre.*



*Following the activity students could be asked to describe boxes having different measurements that would have a volume of one cubic centimetre.*

Distribute copies of BLMs 8 and 9, scissors, and tape. Encourage students to be as accurate as they can in constructing the centimetre cubes so that they will be able to use them to determine volumes of the boxes they will be making. Each pair/group may need more than one copy of BLM 8, depending on how many small cubes they make and how carefully they arrange the figures to be cut. Encourage students to use the technique described in the box above to determine the volume of each constructed box.

*As an added challenge, you may wish to point out that one copy of BLM 8 provides enough squares to make Boxes 1, 2, 3, and 4 as well as eight centimetre cubes using the pattern given. Students should sketch all the figures before cutting them to see if they can fit them all onto one copy of BLM 8. (See Solutions and Notes for one possible arrangement.)*

For problem 6, students should estimate answers. Problem 7 asks them to determine accurate answers.

Numbers 6 and 7 ask students for surface area. They will be familiar with “area” but may not have encountered surface area of a three-dimensional figure before. Explain to them that this just means the number of square centimetres there are on the surfaces of the box (the bottom and four sides). You may wish to have students add the area of the top as well, even though the constructed boxes do not have tops. Students should see that the top will have the same area as the bottom. Note that, in #7, because the boxes have been constructed out of squared paper, students can simply count the number of square centimetres on each surface of the box to determine the surface area.

A porcupine is 76 cm long with a 15 cm tail. It weighs 7-11 kg. Design a box to hold an adult porcupine.

### Problem Solving





### Activity 3: Inside and Outside Boxes

If time permits, repeat the activity with boxes made from 16-cm by 16-cm squares. Have each group make only 1 or 2 of the boxes. There will be seven boxes as in the table below.

*You may wish to give students the first 3 or 4 lines of the table and ask if they see any patterns in the measurements. For example, the dimensions of the base decrease by 2 cm each time; the box number is the same as the number of centimetres in the height.*

Box	Dimensions of base	Height
1	14 cm by 14 cm	1 cm
2	12 cm by 12 cm	2 cm
3	10 cm by 10 cm	3 cm
4	8 cm by 8 cm	4 cm
5	6 cm by 6 cm	5 cm
6	4 cm by 4 cm	6 cm
7	2 cm by 2 cm	8 cm

When samples of all the boxes have been constructed, have students estimate relative volumes. Ask students to identify the box they think has the greatest volume and which has the least. Have them put the boxes in order of size.

#### Extensions in Mathematics:

1. Examine some of the boxes in the class collection that give the mass of the contents. Ask students if there is any relationship between the size of the box and the weight of its contents. For example, in one sample, a box of cereal weighed 350 g, while a smaller box of cake mix weighed 510 g, and a box of crackers (between the other two in size) weighed 200 g. Ask students why a small box might be heavier than a large box. Give them time to consider and then share their reasons with the class. Have them support their theories with examples.
2. Explore volumes of particular cylinders using BLM 10. This asks students to compare volumes of cylinders by estimation, but does not ask for the actual volume. Since none of the cylinders will have a bottom, measuring the volume or even comparing volumes by pouring material from one cylinder to another raises a problem. Questions 3 to 7 ask students to describe a method they might use to see which of two cylinders holds more. Discuss these methods with students, and use one or two to determine if their comparisons were correct.

If you want students to determine the actual volumes of the cylinders, measuring (calibrated) bottles can be made by students. See BLM 11, which could even become a Family Activity for students to do at home as indicated below.

See page 3 for a description of “Rules of Thumb.”

#### R of T

There are approximately 40 mice to the litre. If this is true, how much space does a mouse take up?

#### Problem Solving



**Activity 3: Inside and Outside Boxes****Family Activities:**

1. Send home copies of BLM 11 and have students make a “measuring bottle” marked at 125 ml, 250 ml, 375 ml, ... 1000 ml (or 1 L).

**Other Resources:**

For additional ideas, see annotated Other Resources list on page 54, numbered as below.

3. By the Unit or Square Unit?, by B.B. Ferrer.
10. Food for Thought, by G.C. Poke, N. Ewing, and D. Stevens.



## Activity 4: Hot and Cold

### Focus of Activity:

- Observing and measuring temperature changes under several conditions
- Identifying conditions that affect temperature
- Collecting and graphing of data

### What to Assess:

- Accuracy of measurements
- Reasonableness of predictions
- Accuracy of graphs
- Reasonableness of explanations of why temperature changes vary under different conditions

### Preparation:

- Collect 4 to 6 identical bottles, pieces of various materials large enough to wrap around the bottles, and elastics to hold them on, and a Celcius thermometer.
- Provide a thermometer in Celcius degrees for each group.
- Make copies of BLM 12 for each group.
- Provide a container of ice for each group.
- Provide graph paper.

### Activity:

*This cross-curricular activity touches on several topics in Science, such as*

- using a thermometer to measure temperature
- testing insulating materials
- compiling data and using graphs and tables
- minimizing heat loss

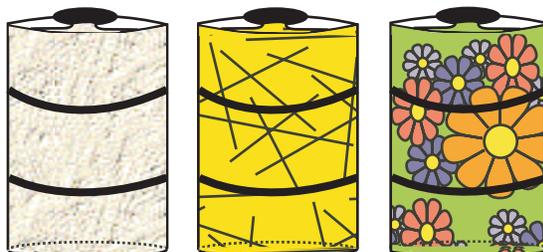
### TESTING BOTTLES

Discuss with students how we dress for different temperatures, and how houses are insulated against cold. List materials they think are good insulators. Show the materials collected and have students indicate which they think will be good insulators and which will not.

A suitable collection of materials might include:

- |           |                                   |
|-----------|-----------------------------------|
| wool      | aluminum foil                     |
| felt      | several layers of newsprint       |
| cotton    | quilt batting                     |
| fake fur  | synthetic fabric such as nylon    |
| towelling | plastic (e.g., green garbage bag) |

Run a hot water tap until maximum temperature is achieved, and fill each bottle. This is a way of ensuring that all bottles initially have water of the same temperature. Wrap each bottle in a different material. Fasten on with elastics.



The temperature in a jungle at ground level can be 39°C; the humidity can be 90%.



### Activity 4: Hot and Cold

Take the temperature of the water in each bottle, and record at 0 minutes in a chart such as the one below:

Bottle Covering	0 min	15 min	30 min	60 min	75 min
Aluminum Foil					
Felt					
Towelling					

Assign 3 or 4 students the task of taking and recording the temperature every 15 minutes. If you have a timer, this can be used as a reminder. Students may need to take readings during recess or at lunch time.

Ask students what they think will happen eventually, and why. They should recognize that the bottles should all have the same temperature if left long enough.

While this experiment is under way, start the following activity.

#### ICE BUCKETS

Distribute copies of BLM 12 to each group. Read it over with the students so that they understand what they are to do. Since the temperature readings are taken every five seconds, they need to be well organized. Suggest one student be responsible for reading the times (e.g., “10 seconds ... 20 seconds ... 30 seconds ...”); another student reads the thermometer, saying the temperature out loud, while another student records the temperatures in the chart on BLM 12.

Have students predict what they think will happen. Record some predictions to be compared with the actual results.

Distribute buckets of ice. Have students put thermometers in the ice and record the temperature in the first row of Table 1. For the next four minutes, they need to take several readings. Then they should remove the thermometer from the ice and lay it on a desk, recording its present temperature in row 1 of Table 2. They continue taking readings as the thermometer warms up.

Distribute graph paper and have students graph the data from each table. If some review is necessary, you might wish to discuss with them what numbers should be on each axis. For example, the axes on page 19 could be used for both graphs. Have students graph the data from both tables on the same set of axes.

After the graphs are completed, have students compare the two sets of data.

How are they alike?

How are they different?

Did the temperature change at the same rate for the whole 90 s? How can you tell this from the graph?

Do your graphs differ from those of other students? Why might this happen?

#### Communication



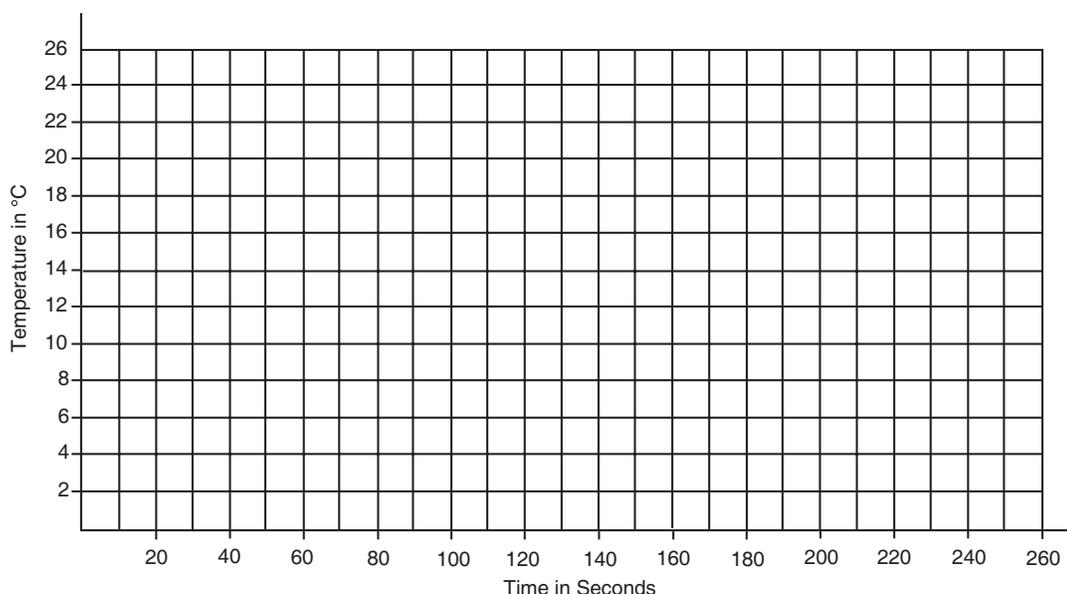
For more on graphs see “Investigations in Data Management”, Gr. 6. Ordering information can be found on the last page of this booklet

#### Problem Solving



### Activity 4: Hot and Cold

Students should realize that where the graph is steepest, the temperature was changing most quickly.



OPTIONAL: Since the experiment takes very little time, you may wish to repeat it using crushed ice, or ice with salt. Ask students what they think will be the effect of these as well as other changes such as the number or size of cubes, and stirring the ice during the experiment. Each group could then select one of these experiments to carry out. They will need additional copies of BLM 12 to record the results.

#### RETURN TO TESTING BOTTLES

When temperatures have been taken over a period of 90 to 120 minutes, have students examine the table. Compare results with their predictions as to which are the better insulators. Ask them why some materials such as fur or aluminum foil are good insulators. They should realize that fur traps lots of air spaces and air is a good insulator. Aluminum foil, on the other hand, reflects heat. Ask if they think it would make a difference if the shiny side were facing the bottle or facing the outside.

See “Solutions and Notes” for sample data.

Other versions of this experiment might involve testing such coverings as different colours of cloth, different thicknesses of fabric, or whether or not the bottles are sitting in the sunlight or in shadow. Alternatively, see Cross-Curricular Activity #1 below.

#### Cross-Curricular Activity:

1. Place several different colours/types of paper or fabric on a sunny windowsill. Under each covering place a thermometer initially at room temperature. Have students read and record the temperature on each thermometer after 5 minutes, after 15 minutes, and after 25 minutes.



Approximate daytime temperatures:  
 Mercury:  $410^{\circ}\text{C}$ ;  $\square$   
 Mars:  $-125^{\circ}\text{C}$  to  $30^{\circ}\text{C}$ ;  
 Saturn:  $-180^{\circ}\text{C}$ ; and  
 Pluto:  $-130^{\circ}\text{C}$ .

#### Problem Solving





The Canadian husky has hair 10-15 cm long over a coat of oily matted wool 5-8 cm long.

#### R of T

One centimetre of rain would make 10 cm of snow. How many centimetres of rain would make 30 cm of snow?

### Activity 4: Hot and Cold

2. Have students research data on animal coats. For example, what is unique about a polar bear's coat that makes it able to withstand the very cold temperatures of the Arctic?

#### Family Activities:

1. Have students gather data on average temperatures in other countries. In families of new Canadians, have students ask family members to describe ways of dealing with extremes of temperature in other countries.

#### Other Resources:

For additional ideas, see annotated Other Resources list on page 54, numbered as below:

15. Weather Watch, by Valerie Wyatt.

## Activity 5: A Mini-Olympics



### Focus of Activity:

- Estimating, measuring, and collecting data while undertaking measurement activities in a game environment

### What to Assess:

- Reasonableness of estimates
- Accuracy of measurements
- Cooperative behaviour
- Correct use of mathematical language
- Analysis of events

### Preparation:

- Make copies of BLMs 13, 14, 15, 16, 17 for each team.
- Make copies of BLM 18 (optional).
- Provide materials needed for any Mini-Olympic Events you plan to use.

### Activity:

This activity provides instructions, on BLMs 13, 14, and 15, for eight different tasks which involve various types of measurement. Students are asked to estimate and then measure using specific units. The units involved are:

- |               |                      |
|---------------|----------------------|
| - millilitres | - square centimetres |
| - angles      | - centimetres        |
| - seconds     | - metres             |
| - decimetres  |                      |

These are intended as a review of measurement topics. Students should work on them in groups, recording their estimates and measurements, and then determining their scores, which depend on the accuracy of their estimates.

Tell students they are going to be taking part in mini-Olympic Games as members of teams. Each team will participate in up to 8 events.

Ways this might be organized are:

1. Have all students participate in 2 or 3 selected events.
2. Have each event set up in a different part of the room, and have the teams cycle through them. This may involve all eight events or only some of them.
3. Make 1 copy of each of BLMs 13, 14, 15; mount the instructions for each event on a card and leave it at the event area rather than giving copies of BLMs 13, 14, and 15 to all groups.

If all 8 events are set up, and teams of 4 are formed, there may be 1 or 2 events not being used at any one given moment. Teams that finish an event quickly will then have another event to begin immediately.

Each event includes a “QUESTION” that asks students to analyze the event in some way. Students could work on those while waiting for an event to become available. These



## Assessment



## Activity 5: A Mini-Olympics

questions could also be considered when students have completed the events. Each team will need a score card for each event. These can be found on BLM 16. Students should realize that, for each event, they make an estimate and then a measurement, and then subtract one from the other to achieve the score for each person. The closer the estimate is to the actual measurement, the lower will be the score. Thus, the winning team will be the one with the lowest score.

As a team progresses through the activities, each successive member should try to use the results of the earlier player(s) to refine his/her estimate. Observe students to see if they are doing this, and suggest it if they are not. In order to give all participants opportunities to do this, a different student should be first at each event.

Most of the materials should be available in the classroom. A full list is given here.

- Event 1: Make A Fist: a 1 L bottle (see below), a basin, water, towels
- Event 2: Footprints: newsprint, acetate centimetre grid (see below)
- Event 3: Seconds To Go: metre stick, clock/watch
- Event 4: Finding the Angles: protractor
- Event 5: Paper Throw: sheets of 8" x 11" paper, ruler/tape
- Event 6: Trundle Along: sheets of 8" x 11" paper, 1-dm trundle wheel (see below)
- Event 7: Cotton Lift: cotton balls, metre tape, straws
- Event 8: Button Snap: a few medium to large buttons, ruler

### Notes on Materials:

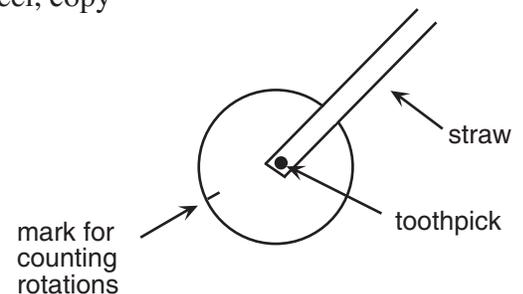
Event 1: If students made measuring bottles earlier (Activity 3, BLM 11), then use one of these.

Event 2: Make a few acetate copies of BLM 8.

Event 6: To make a 1-dm trundle wheel, copy BLM 17, cut out one of the "wheels" shown, and mount it on bristol board. Attach it to a straw with a toothpick.

Alternatively, print one or two copies of BLM 18 on card stock. This will be strong enough so it needn't be mounted on bristol board. You may wish each student to make a trundle wheel. If this is the case you might wish to do Event 6 (Trundle Along) with the whole class.

Event 7: Each student should have his/her own straw, and his/her own cotton ball. The cotton wad from the top of a pill bottle is a good size.



If each team takes part in all 8 Events, then each team will need 2 copies of BLM 16 containing the score cards. Have students write their names in the proper slots. Make sure each team has a beginning event. Let the Games begin!!

## Activity 5: A Mini-Olympics

Observe students participating in Event 5. What do they do to the paper to make it go further? (e.g. crumple it; make a paper airplane). The technique really doesn't matter since the score is not the distance travelled, but the difference between the initial estimate and the actual measurement.

Students participating in Event 6 may be surprised to find that a path around the outside edge of a sheet of paper is very close to one metre long. The metric measurements of an 8" x 11" sheet are 21.5 cm and 28 cm. Thus, the perimeter of the paper is 99 cm.

After completing as many events as you have organized or as time permits, discuss students' responses to the "QUESTION" given with each event. Answers for these would be appropriate for a student's assessment portfolio.

### Cross-Curricular Activities:

- Olympic records are good sources of measurement data. One source where students can find information is the local library. They can mark out some distances in the school yard to become more familiar with measurement units. For example, just how far can an athlete throw a discus? (and what does it weigh?); how high do pole vaulters leap? (measure this up the side of the school); how far does a javelin thrower toss the spear? (and how long is the spear?).

### Family Activities:

- Students could take home instructions for some of the Events and try them with family members. Are adults any better at estimating than the students?

### Other Resources:

For additional ideas, see annotated Other Resources list on page 54, numbered as below.

- What Is A "Good Guess", Anyway? by F.K. Lang.
- Rubber-Band Rockets, by M.E. Hynes, J.K. Dixon, and T. Lott Adams.



The snail travels at a speed of about 2.5 cm/min. Could the snail win Event 3?

### Assessment



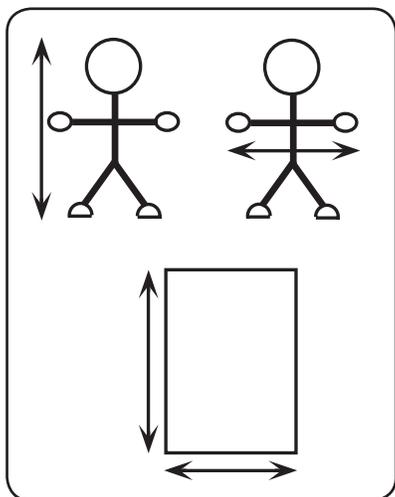
In 1960, Abebe Bikila of Ethiopia set a record in the Olympic marathon of about 5.3 m/s. He ran barefoot!

## BLM 1: What Shape Are You In?

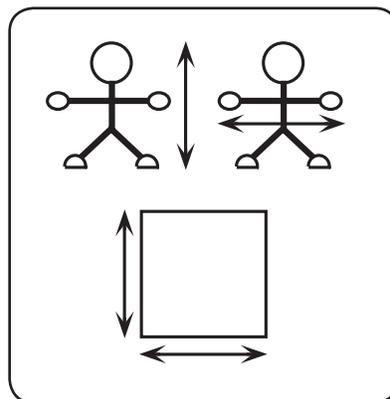
1. (a) Are you a square or a rectangle? To find out, measure, to the nearest centimetre, (i) your height, and (ii) your arm span.
- (b) Compare these measurements. Are they the same or different? If they are different, which one is greater?



The diagrams below show a person who is a rectangle and a person who is a square. Which are you?

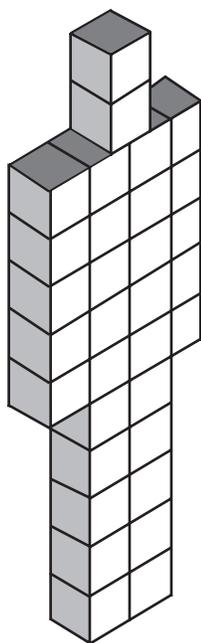


A rectangle



A square

- (c) How many people in your class are rectangles? How many are squares?



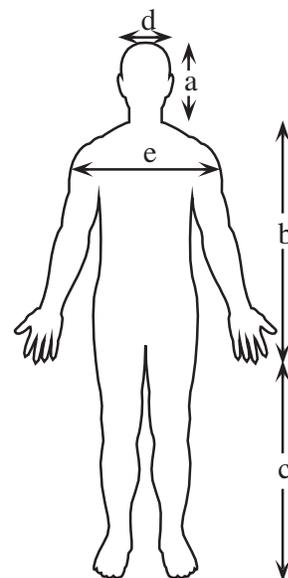
Charlie Cube

2. (a) Imagine you were made of cubes using exactly the same number of cubes as Charlie Cube on the left. How big do you think each cube would be for you? Why?
- (b) Measure from your shoulders to the top of your head. If your head were 2 cubes tall, like Charlie's, how tall would each head cube be?
- (c) Measure your arm from shoulder to fingertips. If your arm were 5 cubes long, like Charlie's, how long would each arm cube be?
- (d) Is your answer to (b) the same as your answer to (c)? Should it be? Explain.
- (e) If Charlie is 144 cm tall, how tall must each cube be?
- (f) If each cube is 12 cm tall, then the volume of each cube (the space it takes) is 1728 cubic centimetres ( $1728 \text{ cm}^3$ ). Use a calculator to determine the total volume of Charlie's body in cubic centimetres.

## BLM 2: Getting in Shape

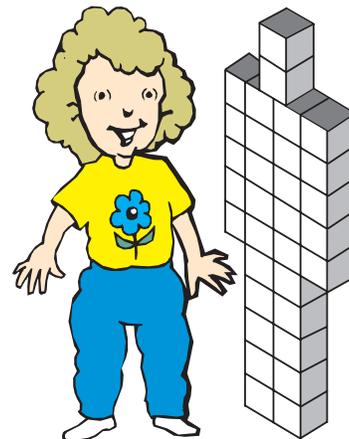
1. Measure each of the following to the nearest decimetre (1 dm = 10 cm):

- the distance from your shoulders to the top of your head \_\_\_\_\_
- the distance from your shoulders to your fingertips \_\_\_\_\_
- the distance from your fingertips to your feet, when your arms are straight down at your sides \_\_\_\_\_
- the width of your head from ear to ear \_\_\_\_\_
- the width of your shoulders \_\_\_\_\_



2. If each cube of Charlie Cube on “What Shape Are You?” were a decimetre cube (a cube 10 cm wide, 10 cm tall, and 10 cm thick) how tall, in decimetres, would Charlie be?

- from his shoulders to the top of his head? \_\_\_\_\_
- from his shoulders to his fingertips? \_\_\_\_\_
- from his fingertips to his feet? \_\_\_\_\_
- from ear to ear? \_\_\_\_\_
- across his shoulders? \_\_\_\_\_



3. Compare your measurements in #1 to Charlie’s measurements in #2.

4. (a) The ratio of Charlie’s measurements in #2 (a), (b), and (c) is 20 to 50 to 50 or 2 to 5 to 5 (or 2:5:5). What is the ratio of your measurements from #1 (a), (b), and (c)? \_\_\_\_\_

(b) What is the ratio of the width of your head to the width of your shoulders? \_\_\_\_\_

(c) Use the ratio of your measurements to design a body made of cubes that is closer to your own body measurements than Charlie Cube is. Sketch “Cubic Me” on a sheet of paper, or on triangle dot paper, or build a model if cubes are available.

(d) How many cubes would be needed to build Cubic Me? \_\_\_\_\_

(e) A decimetre cube has a volume of 1000 cubic centimetres (1000 cm<sup>3</sup>). What is the volume of Cubic Me in cubic centimetres? \_\_\_\_\_

5. **A Challenge:** Write answers on the back of this page.

(a) Do you think your estimate in part (e) is greater or less than the total volume of your body? Why?

(b) Using Cubic You, estimate the total area of skin on your body.

# BLM 3: A Decimetre Ruler

one decimetre	TAB
---------------	-----

one decimetre	TAB
---------------	-----

one decimetre	TAB
---------------	-----

one decimetre	TAB
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one decimetre	TAB
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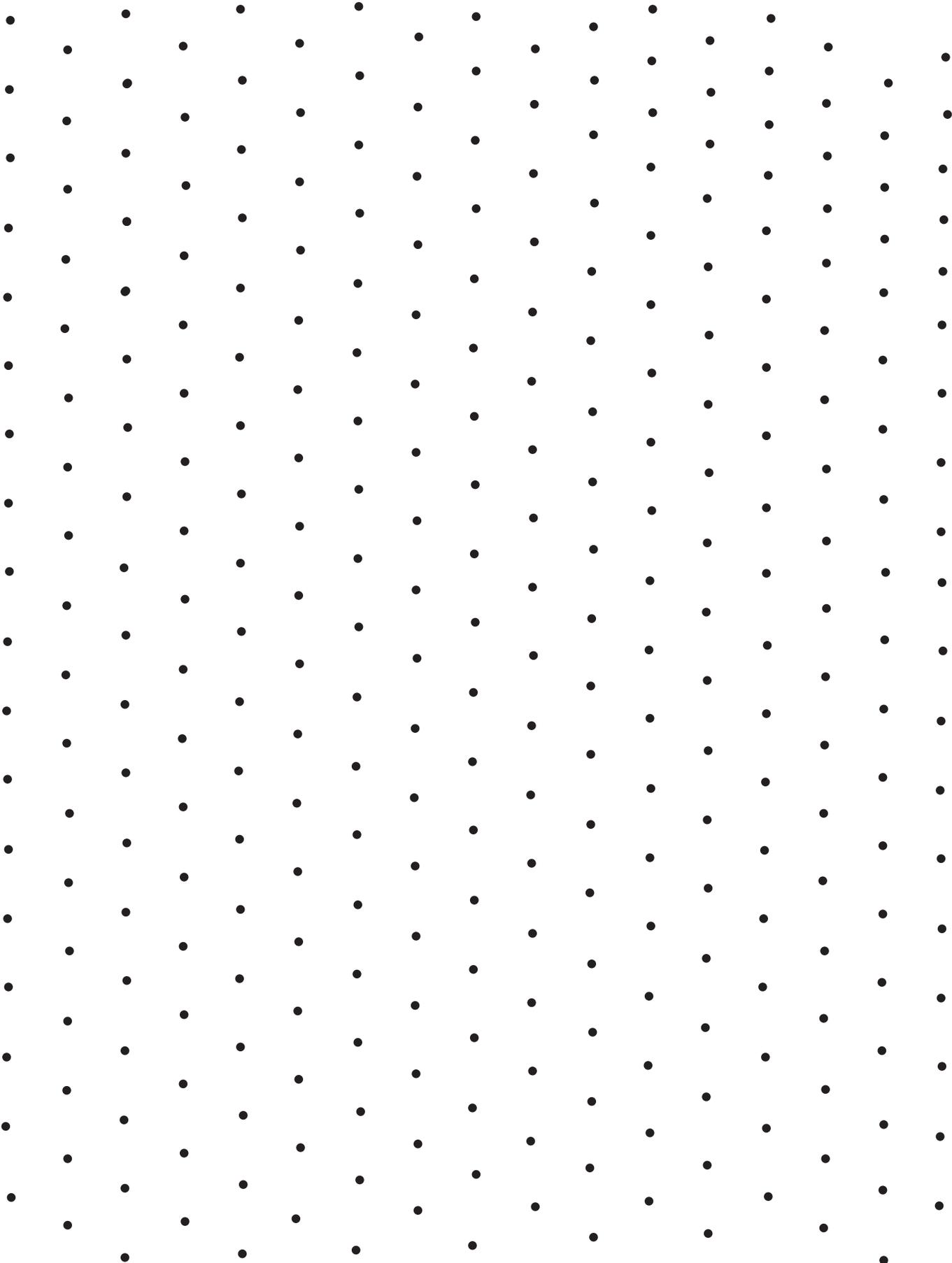
one decimetre	TAB
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one decimetre	TAB
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one decimetre	TAB
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one decimetre
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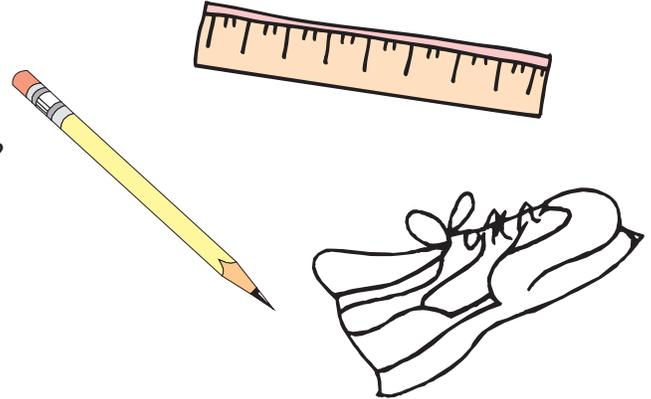
# BLM 4: Triangle Dot Paper



## BLM 5: Thinking About Holes

1. How big a hole do you think you can cut in a sheet of paper the size of this one and still leave at least 1 cm around the outside as a frame?
2. Would the hole be big enough
 

(a) for a pencil to pass through?	(b) for your fist?
(c) for your shoe?	(d) for your math book?
(e) for your ruler?	(f) for your head?
(g) for you?	(h) for your desk?



Tell why you think your answers are correct. Check by cutting the largest hole you can in a sheet of paper.

3. If you had two sheets of paper taped together, would this change any of your answers to #2? Explain. Check by taping two sheets of paper together and cutting the largest hole you can.
4. Which of these do you think would fit through the biggest hole you cut in your two-sheets-together paper? Why do you think so?



## BLM 6: Bigger Than You Think

1. (a) Start with a sheet of copy paper. Carefully measure and cut off a strip so the paper is 20 cm by 28 cm. See Figure 1.

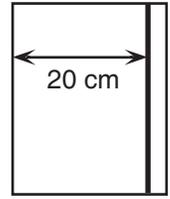


Figure 1

- (b) Fold the paper in half. See Figure 2.



Figure 2

- (c) Measure and draw a line 1 cm from one edge as shown by the heavy line in Figure 3.

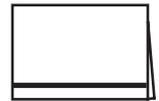


Figure 3

- (d) Mark points on the folded edge 2 cm apart, starting 1 cm from the edge. That is, the points will be at 1 cm, 3 cm, 5 cm, ... Draw lines from these points to the first line you drew. You should have 10 new lines similar to the heavy lines in Figure 4.

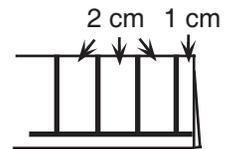


Figure 4

- (e) Without opening the paper, draw a line 1 cm from the folded edge as shown by the heavy line in Figure 5.

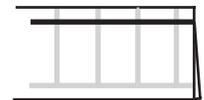


Figure 5

- (f) Draw lines from the long edge that is not folded to this new line. The lines you draw should be halfway between the lines you drew in part (d). You should be drawing 9 new lines, similar to the heavy lines in Figure 6.

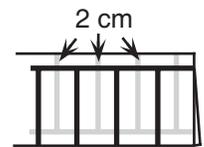


Figure 6

- (g) Carefully cut along the fold from A to B, and then along all the lines you drew in parts (d) and (f) as shown by the heavy lines in Figure 7.

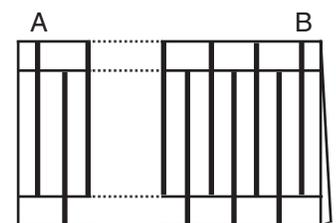


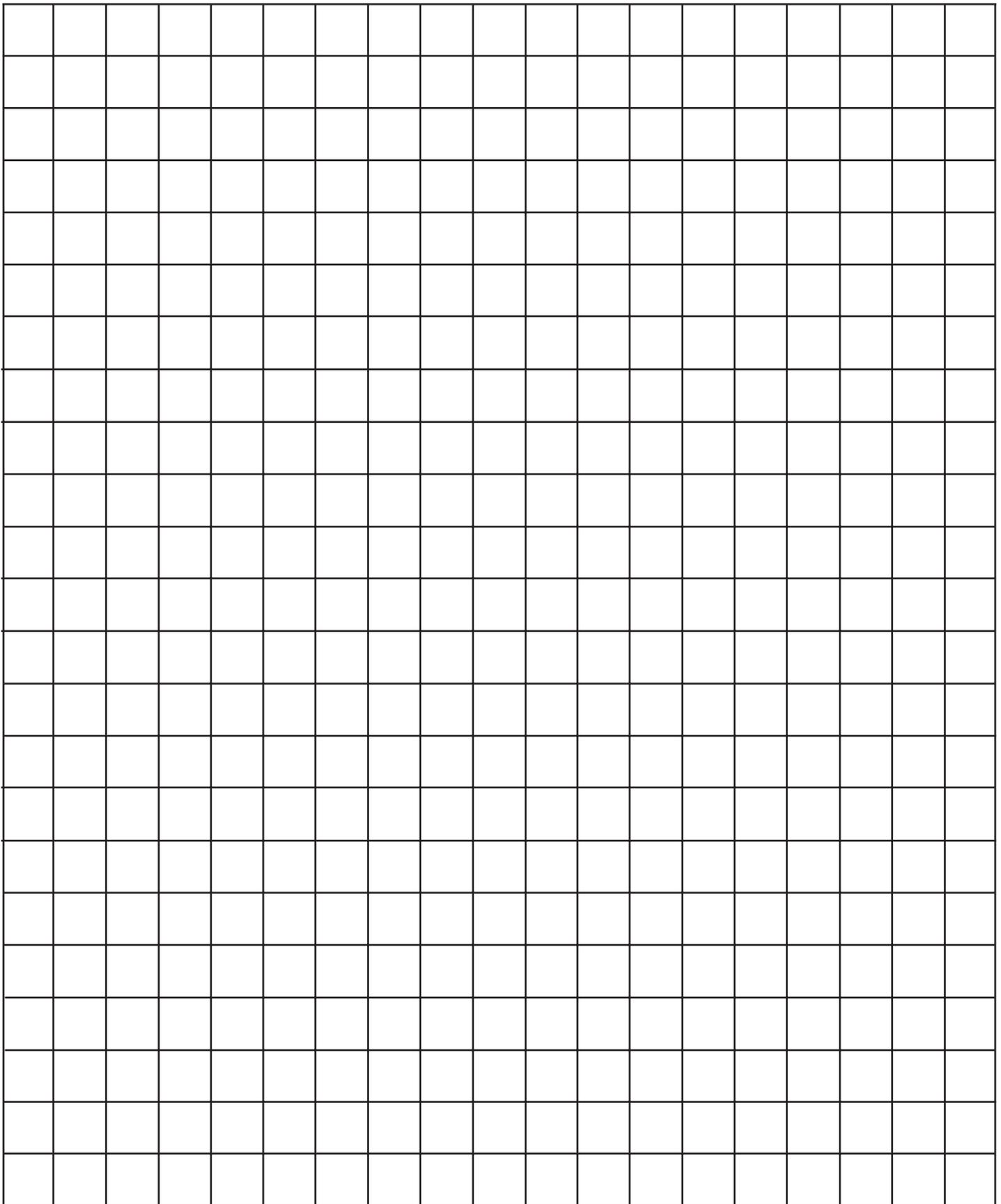
Figure 7

- (h) Carefully unfold the paper. Be careful not to tear it.
- (i) Do you want to change any of your answers for #2 on BLM 5? Why?
2. (a) How many students do you think would fit through the hole? Why?
- (b) If you spread out the paper frame, how many floor tiles do you think it could surround?
- (c) Do you think the hole in the paper is as big as your classroom door? Explain.

**BLM 7: A Template**

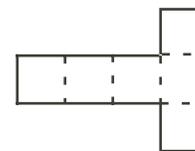
←		
B →		
←	P U T	
←		
←	O N	
←		
←	F O L	
←		
←	D E D	
←		
←	E D G E	
←		
←		
←		
←		
←		
A →		
←		

**BLM 8: Centimetre Grid**



## BLM 9: Making Cardboard Boxes

1. (a) Copy the net of a cube shown on the right onto the grid on BLM 8 so each square of the net is 1 cm by 1 cm. Fold each net on the dotted lines and tape the edges together carefully to make a cube that is 1 cm wide, 1 cm high and 1 cm deep. What is the volume of this cube?



- (b) Make 8 of these cubes. They will help you determine the volume of the boxes you will be making.

2. Cut 4 squares 9 cm by 9 cm from the grid on BLM 8. Cut one centimetre square from each corner of one 9-cm x 9-cm square as shown in Figure 1 below. Fold up and tape the sides of this box as shown in Figure 2. Call this Box 1. Set the box aside.

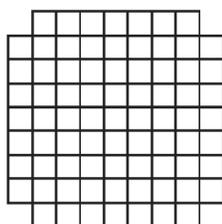


Figure 1

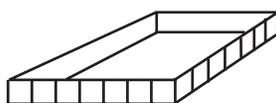


Figure 2

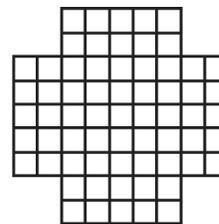


Figure 3

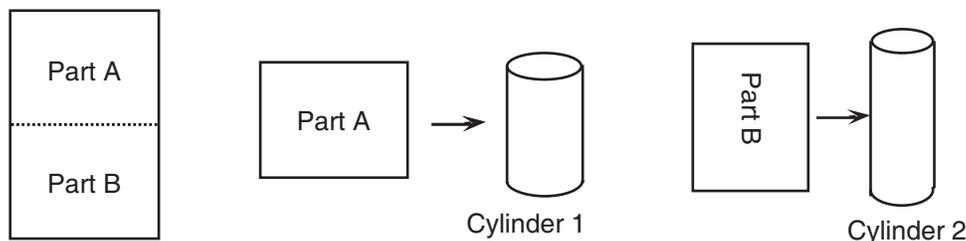
3. From another 9-cm by 9-cm square cut a 2-cm by 2-cm square from each corner as shown in Figure 3 above. Fold up and tape the sides of this box. Call this Box 2. Set the box aside.
4. From another 9-cm by 9-cm square cut a 3-cm by 3-cm square from each corner. Fold up and tape the sides of this box. Call this Box 3. Set the box aside.
5. From the last 9-cm by 9-cm square cut a 4-cm by 4-cm square from each corner. Fold up and tape the sides of this box. Call this Box 4.
6. Look at the four boxes you have made.
- (a) Which one do you think has the greatest volume? Why do you think so?
- (b) Which one do you think has the greatest surface area? Why do you think so?
7. Calculate the volume (in  $\text{cm}^3$ ) and the surface area (in  $\text{cm}^2$ ) for each box and enter these in the table below. You may use your centimetre cubes from #1 to help you determine the volume of each box. What is an easy way to determine the surface area?

Box	Width in cm	Length in cm	Height in cm	Volume in $\text{cm}^3$	Surface Area in $\text{cm}^2$
1	7	7	1		
2					
3					
4					

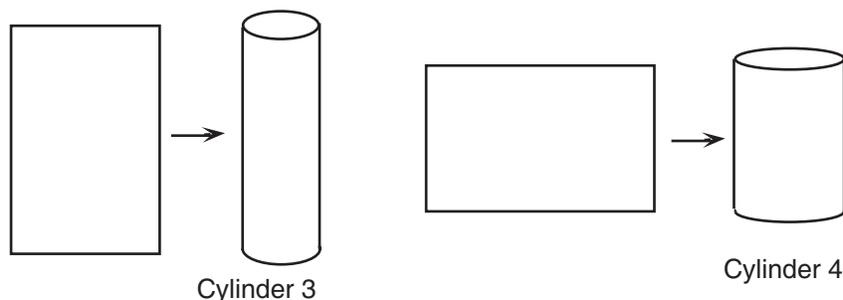
## BLM 10: Cylinders Every Which Way

You will need 4 sheets of paper, scissors, and tape to make the cylinders.

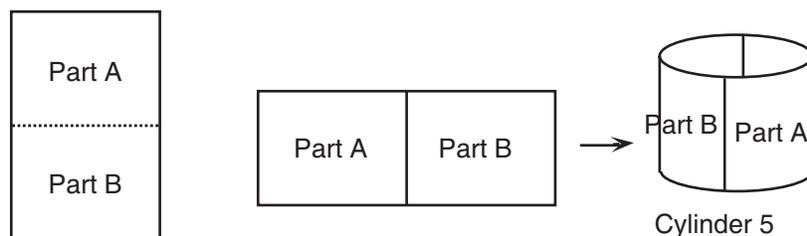
- For Cylinders 1 and 2, cut a piece of paper in half as shown by the dotted line and tape the two parts as shown below to make two different cylinders.



- For Cylinders 3 and 4, use a full sheet of paper each. Tape the edges of one sheet together as shown to make Cylinder 3. Tape the other two edges together For Cylinder 4.



- Compare Cylinders 1 and 3. How are they alike? How are they different? Which do you think will hold more? Why?
- Compare Cylinders 2 and 4. How are they alike? How are they different? Which do you think has the greater volume? Why?
- Compare Cylinders 1 and 2. Which do you think will hold more? Why? How could you test your prediction?
- Compare Cylinders 3 and 4. Which do you think will hold more? Why? How could you test your prediction?
- List the cylinders 1 to 4 in order of size, smallest to largest. Tell why you think your list is correct.
- Cut a piece of paper as shown by the dotted line in the diagram below. Tape the two pieces together as shown to make a cylinder. Call this Cylinder 5.



- Where would the size of cylinder 5 fit into your list from #7?

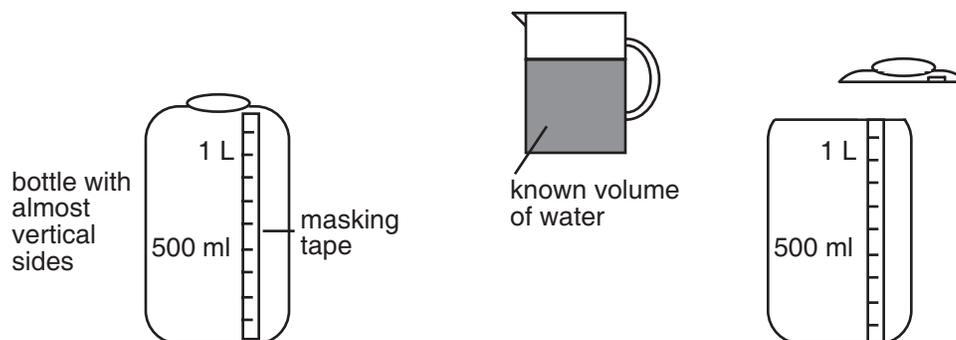
## BLM 11: Making a Measuring Bottle

You will need:

- a large bottle made of clear plastic with sides that are vertical or almost vertical,
- a measuring cup,
- a strip of masking tape, and
- a marker.

When you are identifying water levels, make sure that the bottle and measuring cup are on flat surfaces, and the water has settled.

1. Place the strip of masking tape on the side of the bottle running from top to bottom. (See diagram below.)
2. Measure 125 ml of water in the measuring cup, and pour this into the bottle.
3. When the water has settled, mark the level of the water on the masking tape and label it “125 ml”.
4. Measure another 125 ml of water with the measuring cup, and pour that into the bottle.
5. When the water has settled, mark the level of the water on the masking tape and label it “250 ml”.
6. Repeat to find and mark levels for 375 ml, 500 ml, 625 ml, and so on up to 1000 ml (one litre).  
If your bottle is big enough, you can mark other levels such as 1125 ml, 1250 ml, and so on.
7. If you are going to be using the bottle to measure dry materials such as sand/rice/aquarium gravel, you may wish to cut the top off the bottle. This will make it easier to pour in the dry material.



## BLM 12: Ice Buckets

You will need: a thermometer, a container of ice, and a clock or watch that shows seconds.

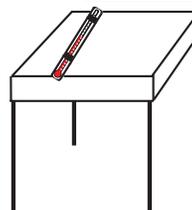
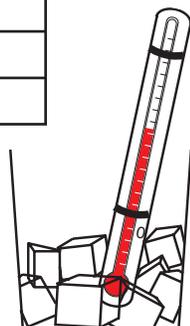
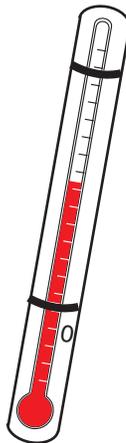
1. Put the thermometer in the ice. Read the temperature and record it in the first row of Table 1. Take the temperature every 10 s, and record in the table.
2. When you have completed Table 1, remove the thermometer from the ice and lay it on a desk. Read the temperature immediately and record it in the first row of Table 2. Take the temperature every 10 s and record in the table.

Table 1

Time in seconds	Temperature in °C
0	
10	
20	
30	
40	
50	
60	
70	
80	
90	
100	
110	
120	
130	
140	
150	
160	
170	
180	
190	
200	
210	
220	
230	
240	

Table 2

Time in seconds	Temperature in °C
0	
10	
20	
30	
40	
50	
60	
70	
80	
90	
100	
110	
120	
130	
140	
150	
160	
170	
180	
190	
200	
210	
220	
230	
240	



## BLM 13: Mini-Olympics, Events 1, 2, and 3

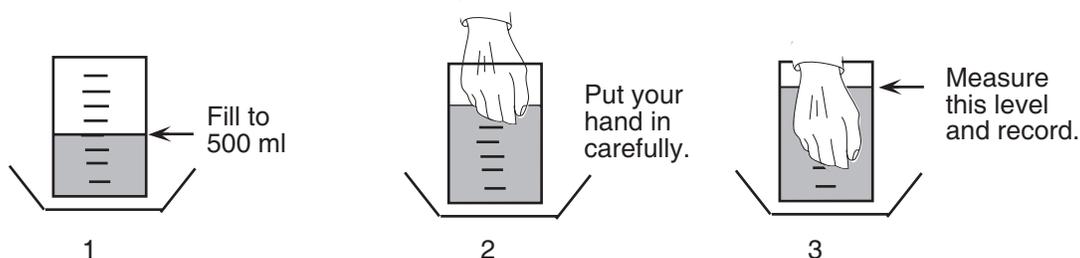
### Event 1: Make A Fist

Materials: a one litre measuring bottle that is large enough for you to put your hand in, a basin, towels, water

Place the litre measuring bottle in the basin. Fill the measuring bottle to the half litre mark with water. Clean up any spilled water.

Estimate how high the water level will rise if you put your fist in the water.

Record this estimate on your score card, and then put your fist in the water.



Measure the water level, record, and figure out how many millilitres of water your hand displaced (pushed aside). Record this on your score card.

Your score is the difference between your estimate and the measure.

QUESTION: Will it make a difference if you use a fist or an open hand? Explain. Do both your hands displace the same amount of water?

### Event 2: Footprints

Materials: plain paper or newsprint; transparent centimetre grid.

Trace around your foot on paper. Estimate the number of square centimetres your footprint covers. Record in on your score card.

Use the transparent grid to count the square centimetres in your footprint.

Record on your score card.

Your score is the difference between your estimate and your count.

QUESTION: Would your footprint area be different if you measured it using a grid in which the lines were just 0.5 cm apart? Explain.

### Event 3: Seconds To Go

Materials: metric ruler, clock with second hand

Mark 2 spots on the floor 5 m apart. Moving at the same speed all the time, try to walk from one to the other in exactly five seconds.

Record how long you actually took.

Your score is the difference between your actual time and 5 s.

QUESTION: Would it be easier or harder to try to walk 5 m in ten seconds? Explain.

## BLM 14: Mini-Olympics, Events 4, 5, and 6

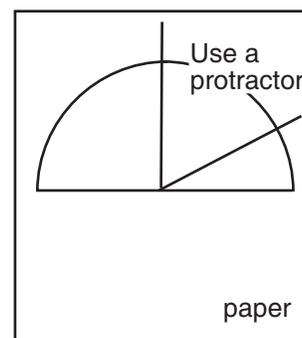
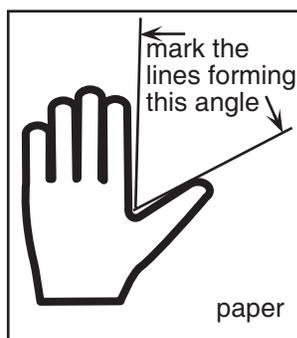
### Event 4: Finding the Angles

Materials: protractor

Lay your hand (either one) on a piece of paper on a flat surface and spread your thumb away from your other fingers. Fingers and thumb should all remain on the flat surface.

Try to form a  $45^\circ$  angle (half of a right angle).

Mark the lines forming the angle as shown.



Remove your hand and use a protractor to measure the angle.

Your score is the difference between your estimate and your measure.

QUESTION: If you could use any parts of your body to form angles, what do you think is the largest angle you could form? What parts of your body would you use? Explain.

### Event 5: Paper Throw

Material: sheets of paper, ruler or tape

How far do you think you can throw a piece of paper? Estimate how far it will go to the nearest decimetre. Record your estimate on your score card.

Throw the paper and measure the distance it travelled to the nearest decimetre. Record this on your score card. Your score is the difference between your estimate and the actual measurement of the distance.

QUESTION: What could you do to the paper to make it go farther? Explain. Test your theory.

### Event 6: Trundle Along

Materials: copy paper/newsprint, 1-dm trundle wheel

On a sheet of paper, draw a pathway about 1 m long. (Your pathway does not need to be straight.)

Using the 1-dm trundle wheel, measure the length of your path.

Your score is the difference between the length of your path and 10 dm (1 m).

QUESTION: When is it easier to use each of the following for measuring:

(i) a ruler? (ii) a measuring tape? (iii) a trundle wheel?

Give reasons for your answers.

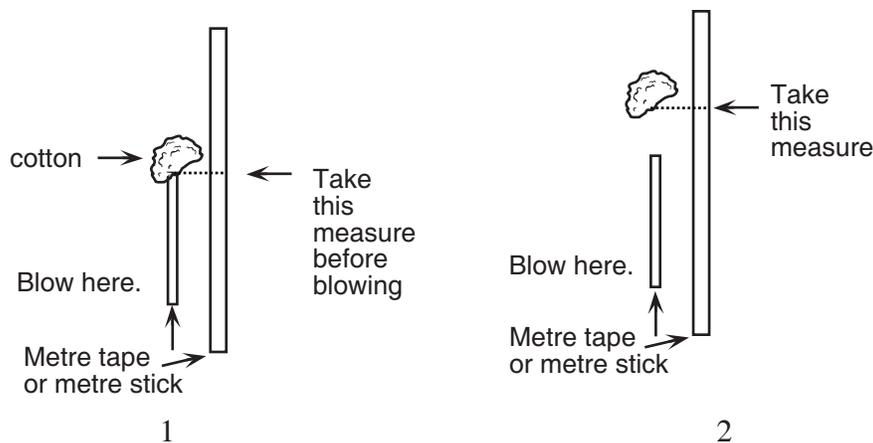
## BLM 15: Mini-Olympics, Events 7 and 8

### Event 7: Cotton Lift

Materials: straws, wad of cotton, metric tape or stick fastened to a vertical surface

Stand beside the metric tape. Hold the straw as shown and place the cotton on top of it. Estimate how far up you can blow the cotton. Record this on your score card.

Before you blow, have a teammate note how high on the tape the cotton is.



Blow the cotton ball into the air, and have a teammate note how high it went. Calculate how far you blew the cotton. Record this on your score card.

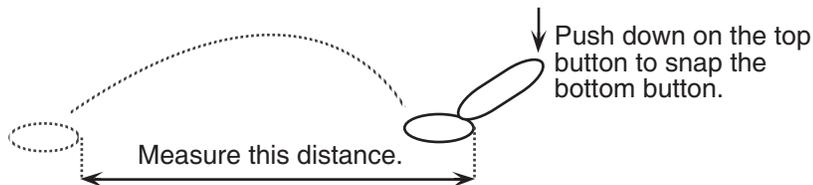
Your score is the difference between your estimate and the measured distance.

QUESTION: Which is more successful: blowing as hard as you can or blowing as long as you can? Why?

### Event 8: Button Snap

Materials: some medium to large size buttons, a meter tape/ruler.

Snap one button with another.



Estimate how far the button will have travelled. Record this on your score card.

Measure how far the button travelled. Record this on your score card.

Your score is the difference between the estimate and the measurement.

QUESTION: Will the type or size of either button make a difference? Explain.

## BLM 16: Mini-Olympics Score Cards

Event #			
Team members	Estimate	Measurement	Score
Total score:			

Event #			
Team members	Estimate	Measurement	Score
Total score:			

Event #			
Team members	Estimate	Measurement	Score
Total score:			

Event #			
Team members	Estimate	Measurement	Score
Total score:			

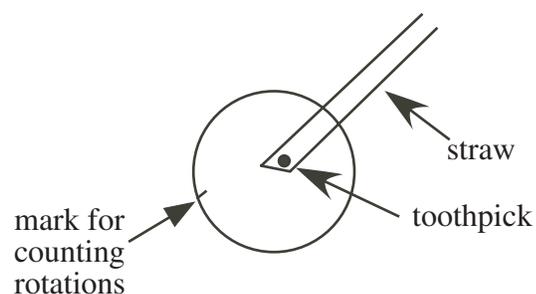
## BLM 17: Decimetre Trundle Wheel

To make a decimetre trundle wheel, you will need the following materials:

- a copy of the trundle wheel (see below).
- a piece of bristol board or a file card
- scissors
- glue
- a short straw (10 - 15 cm)
- a toothpick

Cut apart the circles below labelled “1-dm trundle wheel” and paste each onto some bristol board or a file card. Cut the circle out carefully. You want to keep the circumference (the distance around the circle) as close to 1 dm as possible.

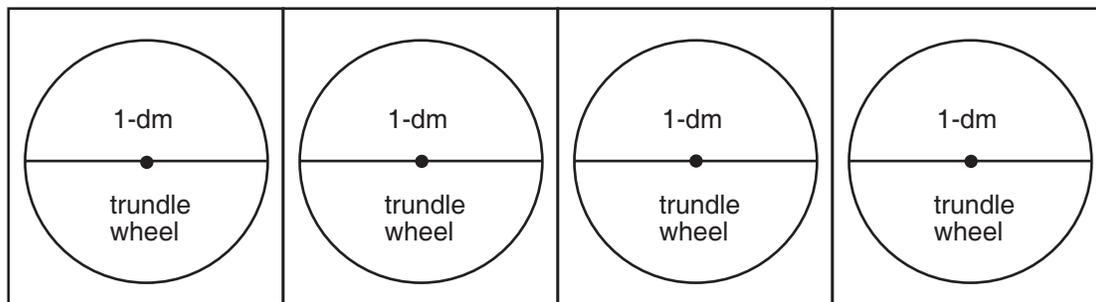
Carefully poke a small hole in the centre of the circle. Push the toothpick through the hole in the centre of the trundle wheel and then through one end of the straw as shown in the diagram below. You may want to break the toothpick and use only a short piece of it.



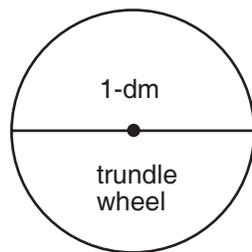
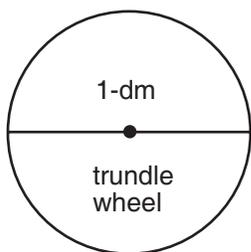
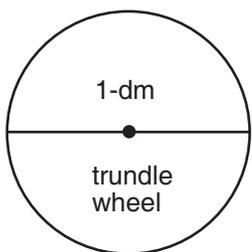
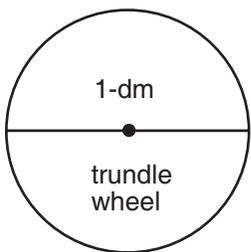
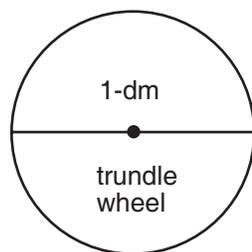
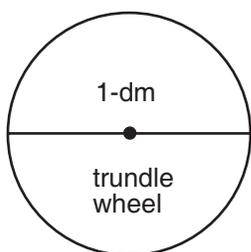
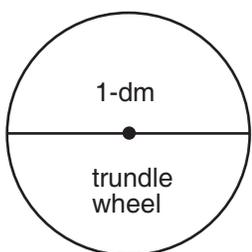
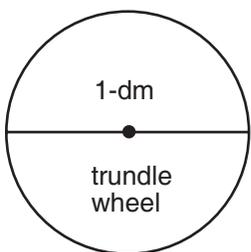
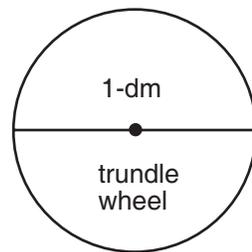
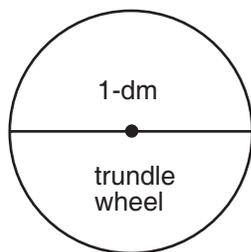
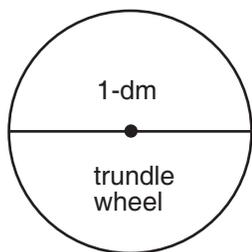
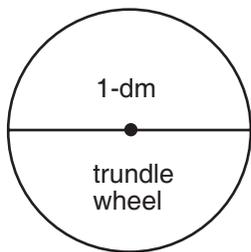
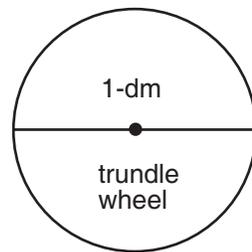
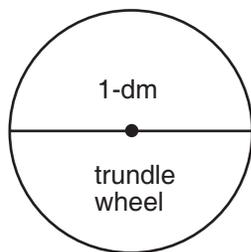
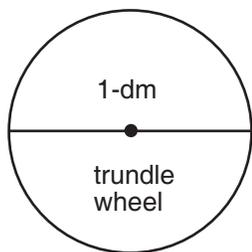
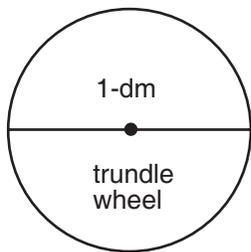
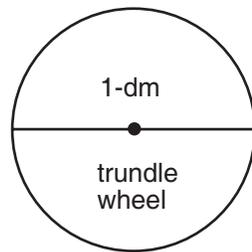
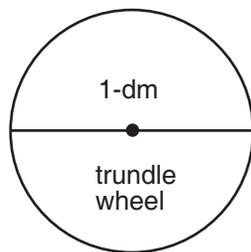
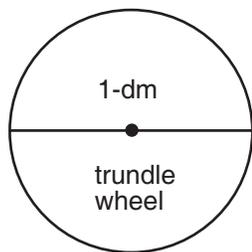
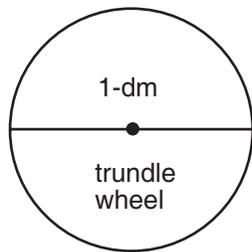
To test its accuracy, draw a 2-dm straight line using a ruler. Roll the trundle wheel along the line. How many times should it go around? Is it accurate? (within 1 mm? within 2 mm?)

Can you figure out a way to make it easy to count the number of rotations?

You might want to put marks on your trundle wheel to show each centimetre. Can you figure out a way to do this accurately?



# BLM 18: Trundle Wheel Templates



Solutions & Notes

Activity 1: Shapes of People

BLM 2: Getting in Shape

4. (c) The diagrams on the right show a “Cube Me” of one of the authors. Figure 1 is a front view, and shows the number of cubes in height and width. Since the depth is one cube, a diagram like this is adequate. If, however, students have had some experience with sketching on isometric (triangle dot) paper, you may choose to have them make a sketch such as Figure 2.

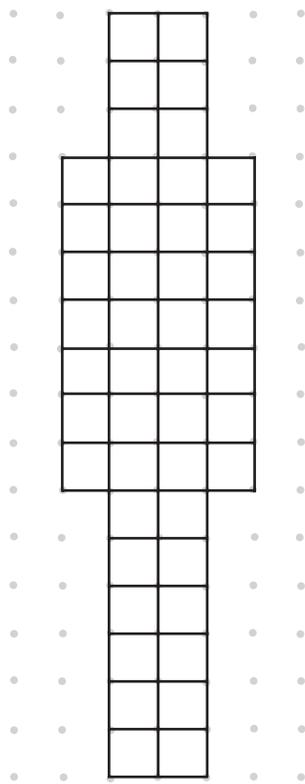


Figure 1

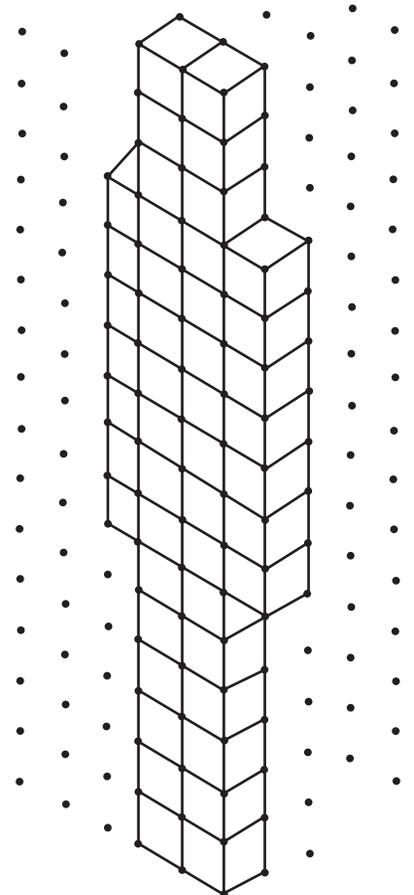
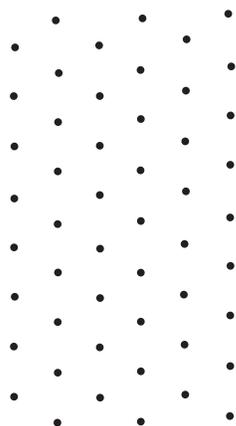


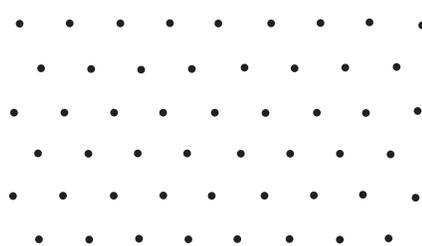
Figure 2

Note that when sketching on isometric paper, the paper is oriented as on BLM 4, so that the dots form vertical lines.

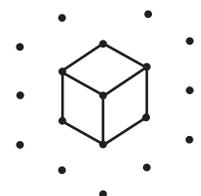
This way:



Not this way:



Each cube can then be drawn as if it is turned so that one edge is toward the viewer.



## Solutions & Notes

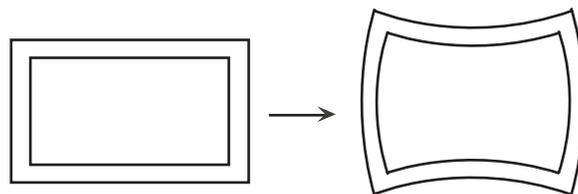
### Activity 2: Holes for Everything

#### BLM 5: Thinking About Holes

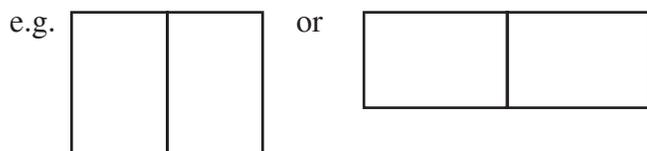
1. Answers should be close to 19.5 cm by 26 cm since the sheet of paper is approximately 21.5 cm by 28 cm.

2. Items a, b, c, d, e will go through a hole in the paper cut as described in #1. If students realize that the 'frame' can be pulled into a rounded shape, they may discover that their heads (f) will fit through the hole.

Items g and h will not fit through such a hole.



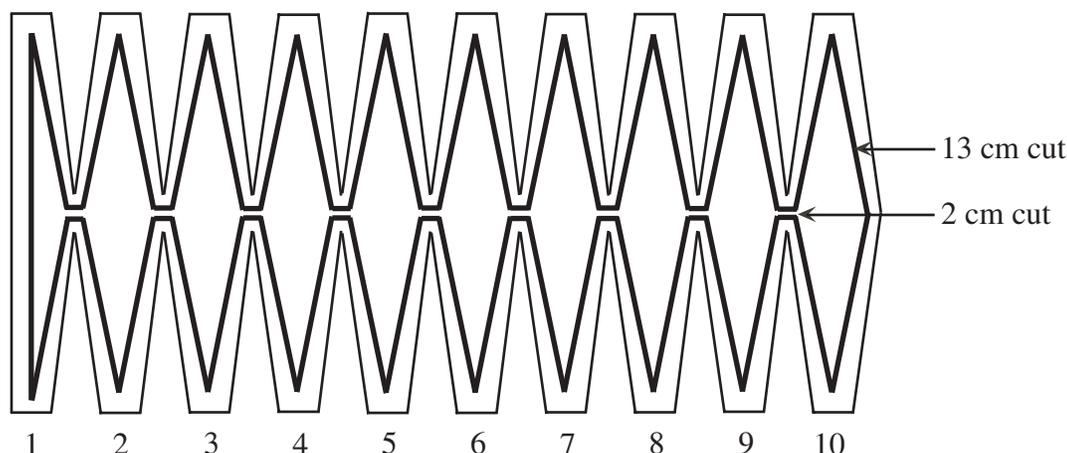
3. With two sheets of paper, students should be able to stretch the 'frame' enough to get their bodies through (#1g). This will be true regardless of which way students taped the two sheets together to begin with.



You might wish to ask students which of these frames has the greater perimeter, and whether or not they can decide this without measuring. Since the length and width of the paper are roughly in the ratio 4:3, we could calculate the perimeter of the first illustration above as  $3 + 3 + 4 + 3 + 3 + 4$  or 20 and of the second as  $4 + 4 + 3 + 4 + 4 + 3$  or 22.

### Extensions in Mathematics

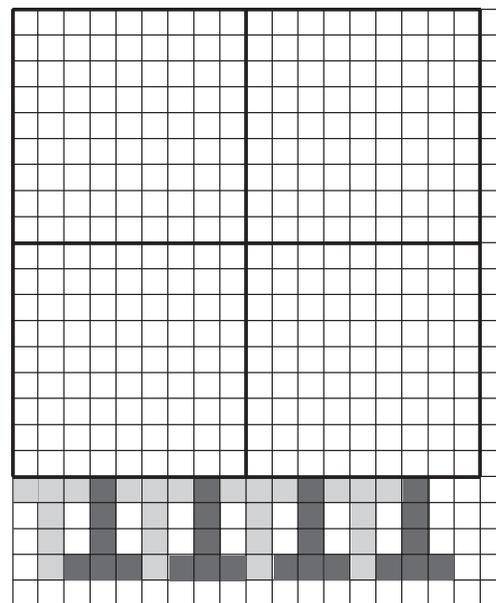
1. The 'size' of the hole can be determined by calculating the inner perimeter. Consider which cuts form the hole. The heavy lines in the diagram below show the relevant cuts. Notice that each 13 cm cut is 'used' four times and each 2 cm cut is 'used' twice to achieve this inner perimeter. There are ten 13 cm cuts and nine 2 cm cuts involved. Thus the 'size' of the hole is 556 cm.



If the paper is folded lengthwise instead of crosswise, there is no need to cut off a strip as done in Step 1 (a) on BLM 6. Therefore the whole sheet of paper is being used. This would suggest that the hole will be larger and, in fact it is 598 cm in perimeter.

**Activity 3: Inside and Outside Boxes**

One way to fit the 4 boxes and 8 cube nets from BLM 9 onto one sheet of paper (BLM 8) is shown.

**BLM 9: Making Cardboard Boxes**

Box	Width in cm	Length in cm	Height in cm	Volume in cm <sup>3</sup>	Surface Area in cm <sup>2</sup>	Surface Area in cm <sup>2</sup> including a top
1	7	7	1	49	77	126
2	5	5	2	50	65	90
3	3	3	3	27	45	54
4	1	1	4	4	17	18

**BLM 10: Cylinders Every Which Way**

One way to compare Cylinders 1 and 2 is to place Cylinder 2 inside Cylinder 1. Then fill Cylinder 2 with rice/sand. Carefully lift Cylinder 2 up, allowing the filling material to remain in Cylinder 1.

If Cylinder 2 holds more than Cylinder 1, the filler will overflow Cylinder 1. If Cylinder 2 holds less than Cylinder 1, the filler will not fill Cylinder 1.

In order of size, smallest to largest, the cylinders are 2, 1, 3, 4, 5.

**Activity 4: Hot and Cold****TESTING BOTTLES**

Partial data from an experiment in which bottles of hot water were wrapped in felt, quilt batting, aluminum foil, and short nap fake fur are given here.

Bottle Coverings	Temperature after				
	30 min.	45 min.	60 min.	75 min.	90 min.
felt	49°C	48°C	44°C	42°C	40°C
quilt batting	53°C	50°C	48°C	44°C	43°C
aluminum foil	54°C	52°C	50°C	48°C	47°C
fake fur	55°C	53°C	50°C	49°C	47°C

## Solutions &amp; Notes

## BLM 12: Ice Buckets

Sample data given below can be compared with student results.

Table 1

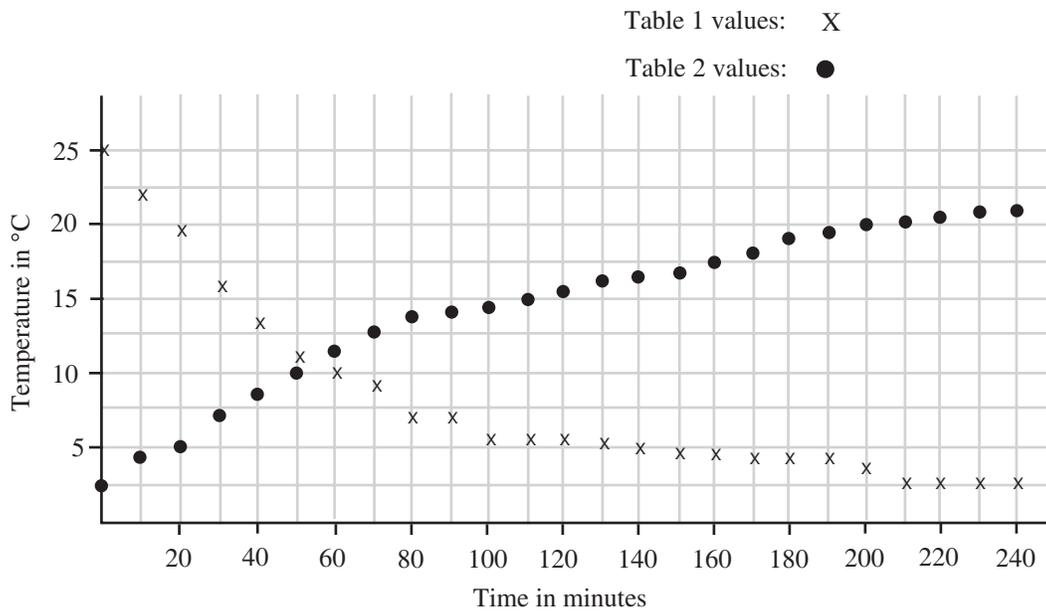
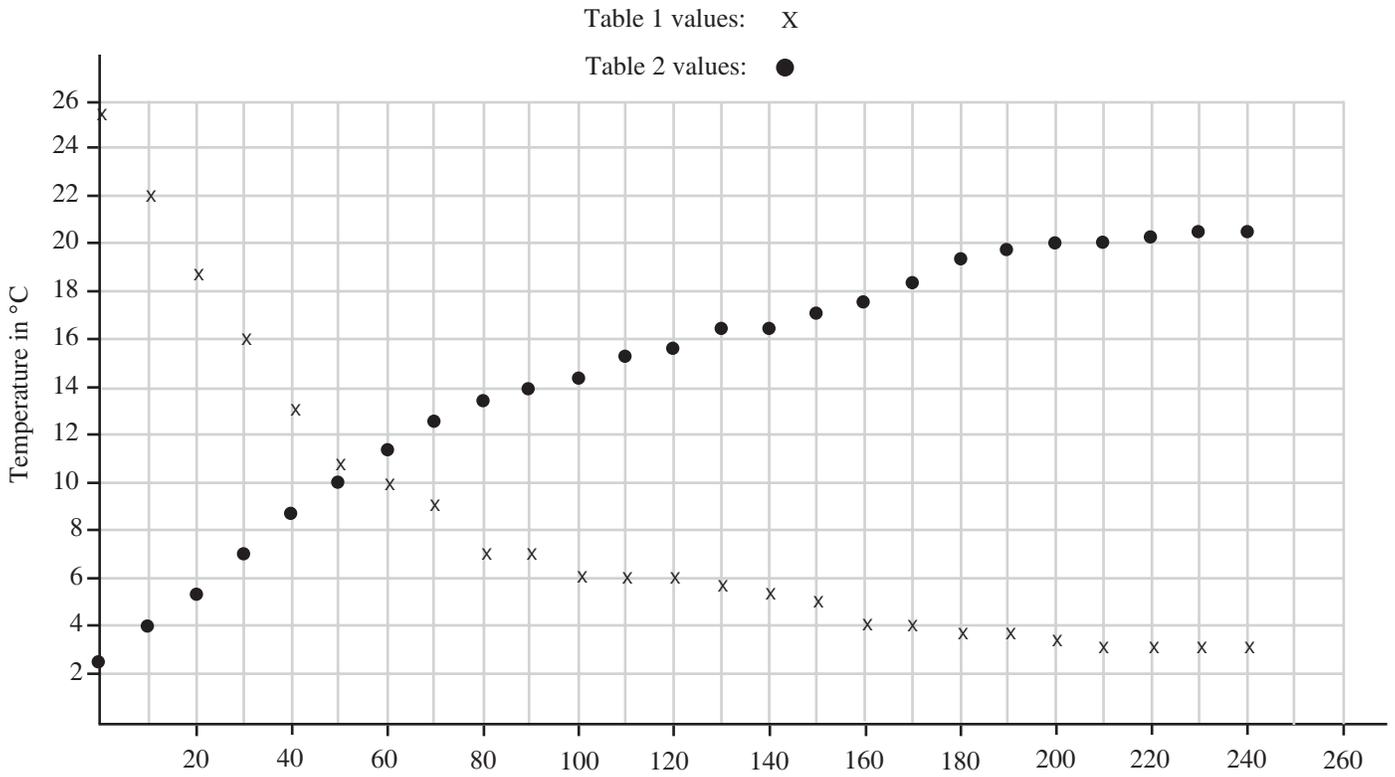
Time in seconds	Temperature in °C
0	25
10	22
20	19
30	16
40	13
50	11
60	10
70	9
80	7
90	7
100	6
110	6
120	6
130	5.5
140	5
150	4.5
160	4
170	4
180	3.5
190	3.5
200	3
210	2.5
220	2.5
230	2.5
240	2.5

Table 2

Time in seconds	Temperature in °C
0	2.5
10	4
20	5
30	7
40	8.5
50	10
60	11.5
70	12.5
80	13.5
90	14
100	14.5
110	15
120	15.5
130	16.5
140	16.5
150	17
160	17.5
170	18.5
180	19
190	19.5
200	20
210	20
220	20.5
230	21
240	21

Students' results will vary from this given data for several reasons. The classroom temperature may vary from the given initial temperature given above. Thermometers may be different qualities, and students may not read thermometers accurately. Air spaces in the ice cubes will affect how fast the temperature falls in Table 1.

Solutions & Notes



Basically, both graphs should change most rapidly initially, and then begin to level off.

## Suggested Assessment Strategies

### Investigations

Investigations involve explorations of mathematical questions that may be related to other subject areas.

Investigations deal with problem posing as well as problem solving. Investigations give information about a student's ability to:

- identify and define a problem;
- make a plan;
- create and interpret strategies;
- collect and record needed information;
- organize information and look for patterns;
- persist, looking for more information if needed;
- discuss, review, revise, and explain results.

### Journals

A journal is a personal, written expression of thoughts. Students express ideas and feelings, ask questions, draw diagrams and graphs, explain processes used in solving problems, report on investigations, and respond to open-ended questions. When students record their ideas in math journals, they often:

- formulate, organize, internalize, and evaluate concepts about mathematics;
- clarify their thinking about mathematical concepts, processes, or questions;
- identify their own strengths, weaknesses, and interests in mathematics;
- reflect on new learning about mathematics;
- use the language of mathematics to describe their learning.

### Observations

Research has consistently shown that the most reliable method of evaluation is the ongoing, in-class observation of students by teachers. Students should be observed as they work individually and in groups. Systematic, ongoing observation gives information about students':

- attitudes towards mathematics;
- feelings about themselves as learners of mathematics;
- specific areas of strength and weakness;
- preferred learning styles;
- areas of interest;
- work habits — individual and collaborative;
- social development;
- development of mathematics language and concepts.

In order to ensure that the observations are focused and systematic, a teacher may use checklists, a set of questions, and/or a journal as a guide. Teachers should develop a realistic plan for observing students. Such a plan might include opportunities to:

- observe a small number of students each day;
- focus on one or two aspects of development at a time.

## Suggested Assessment Strategies

### Student Self-Assessment

Student self-assessment promotes the development of metacognitive ability (the ability to reflect critically on one's own reasoning). It also assists students to take ownership of their learning, and become independent thinkers. Self-assessment can be done following a co-operative activity or project using a questionnaire which asks how well the group worked together. Students can evaluate comments about their work samples or daily journal writing. Teachers can use student self-assessments to determine whether:

- there is change and growth in the student's attitudes, mathematics understanding, and achievement;
- a student's beliefs about his or her performance correspond to his/her actual performance;
- the student and the teacher have similar expectations and criteria for evaluation.

### Resources for Assessment

"For additional ideas, see annotated Other Resources list on page 54, numbered as below."

1. The Ontario Curriculum, Grades 1-8: Mathematics.
2. *Linking Assessment and Instruction in Mathematics: Junior Years*, Ontario Association of Mathematics Educators/OMCA/OAJE, Moore et al., 1996.  
The document provides a selection of open-ended problems tested in grades 4, 5, and 6. Performance Rubrics are used to assess student responses (which are included) at four different levels. Problems could be adapted for use at the Junior Level. Order from OAME/AOEM, P.O. Box 96, Rosseau, Ont., P0C 1J0. Phone/Fax 705-732-1990.
3. *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions*, by Jean Kerr Stenmark (Ed.), NCTM, 1991.  
This book contains a variety of assessment techniques and gives samples of student work at different levels.  
Order from Frances Schatz, 56 Oxford Street, Kitchener, Ont., N2H 4R7. Phone 519-578-5948; Fax 519-578-5144. email: frances.schatz@sympatico.ca
4. *How to Evaluate Progress in Problem Solving*, by Randall Charles et al., NCTM, 1987.  
Suggestions for holistic scoring of problem solutions include examples of student work. Also given are ways to vary the wording of problems to increase/decrease the challenge. A section on the use of multiple choice test items shows how these, when carefully worded, can be used to assess student work.

## Suggested Assessment Strategies

### A GENERAL PROBLEM SOLVING RUBRIC

This problem solving rubric uses ideas taken from several sources. The relevant documents are listed at the end of this section.

#### “US and the 3 R's”

There are five criteria by which each response is judged:

**U**nderstanding of the problem,

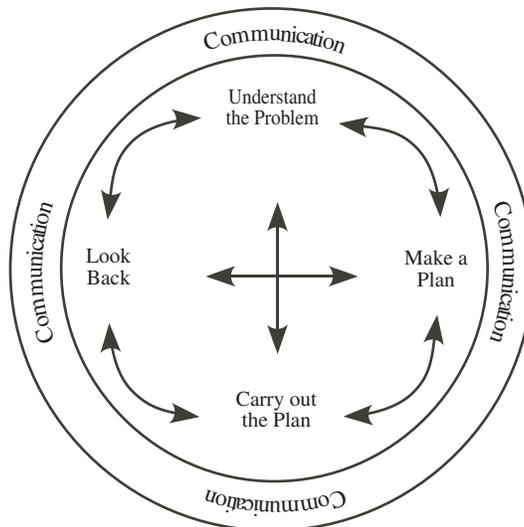
**S**trategies chosen and used,

**R**easoning during the process of solving the problem,

**R**eflection or looking back at both the solution and the solving, and

**R**elevance whereby the student shows how the problem may be applied to other problems, whether in mathematics, other subjects, or outside school.

Although these criteria can be described as if they were isolated from each other, in fact there are many overlaps. Just as communication skills of one sort or another occur during every step of problem solving, so also reflection does not occur only after the problem is solved, but at several points during the solution. Similarly, reasoning occurs from the selection and application of strategies through to the analysis of the final solution. We have tried to construct the chart to indicate some overlap of the various criteria (shaded areas), but, in fact, a great deal more overlap occurs than can be shown. The circular diagram that follows (from OAJE/OAME/OMCA “Linking Assessment and Instruction in Mathematics”, page 4) should be kept in mind at all times.



There are four levels of response considered:

**Level 1: Limited** identifies students who are in need of much assistance;

**Level 2: Acceptable** identifies students who are beginning to understand what is meant by ‘problem solving’, and who are learning to think about their own thinking but frequently need reminders or hints during the process.

**Level 3: Capable** students may occasionally need assistance, but show more confidence and can work well alone or in a group.

**Level 4: Proficient** students exhibit or exceed all the positive attributes of the **Capable** student; these are the students who work independently and may pose other problems similar to the one given, and solve or attempt to solve these others.

## Suggested Assessment Strategies

LEVEL OF RESPONSE 

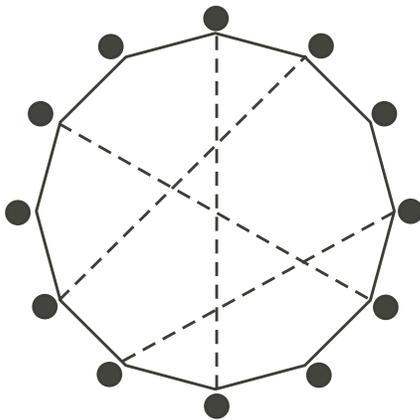
	Level 1: Limited	Level 2: Acceptable	Level 3: Capable	Level 4: Proficient
<b>CRITERIA FOR ASSESSMENT</b> UNDERSTANDING STRATEGIES REASONING REFLECTION RELEVANCE	<ul style="list-style-type: none"> <li>requires teacher assistance to interpret the problem</li> <li>fails to recognize all essential elements of the task</li> </ul>	<ul style="list-style-type: none"> <li>shows partial understanding of the problem but may need assistance in clarifying</li> </ul>	<ul style="list-style-type: none"> <li>shows a complete understanding of the problem</li> </ul>	<ul style="list-style-type: none"> <li>shows a complete understanding of the problem</li> </ul>
	<ul style="list-style-type: none"> <li>needs assistance to choose an appropriate strategy</li> </ul>	<ul style="list-style-type: none"> <li>identifies an appropriate strategy</li> </ul>	<ul style="list-style-type: none"> <li>identifies an appropriate strategy</li> </ul>	<ul style="list-style-type: none"> <li>identifies more than one appropriate strategy</li> </ul>
	<ul style="list-style-type: none"> <li>applies strategies randomly or incorrectly</li> <li>does not show clear understanding of a strategy<sup>1</sup></li> <li>shows no evidence of attempting other strategies</li> </ul>	<ul style="list-style-type: none"> <li>attempts an appropriate strategy, but may not complete it correctly<sup>2</sup></li> <li>tries alternate strategies with prompting</li> </ul>	<ul style="list-style-type: none"> <li>uses strategies effectively</li> <li>may attempt an inappropriate strategy, but eventually discards it and tries another without prompting</li> </ul>	<ul style="list-style-type: none"> <li>chooses and uses strategies effectively<sup>3</sup></li> <li>recognizes an inappropriate strategy quickly and attempts others without prompting</li> </ul>
	<ul style="list-style-type: none"> <li>makes major mathematical errors</li> <li>uses faulty reasoning and draws incorrect conclusions</li> <li>may not complete a solution</li> </ul>	<ul style="list-style-type: none"> <li>may present a solution that is partially incorrect</li> </ul>	<ul style="list-style-type: none"> <li>produces a correct and complete solution, possibly with minor errors</li> </ul>	<ul style="list-style-type: none"> <li>produces a correct and complete solution, and may offer alternative methods of solution</li> </ul>
	<ul style="list-style-type: none"> <li>describes<sup>4</sup> reasoning in a disorganized fashion, even with assistance</li> <li>has difficulty justifying<sup>5</sup> reasoning even with assistance</li> </ul>	<ul style="list-style-type: none"> <li>partially describes<sup>4</sup> a solution and/or reasoning or explains fully with assistance</li> <li>justification<sup>5</sup> of solution may be inaccurate, incomplete or incorrect</li> </ul>	<ul style="list-style-type: none"> <li>is able to describe<sup>4</sup> clearly the steps in reasoning; may need assistance with mathematical language</li> <li>can justify<sup>5</sup> reasoning if asked; may need assistance with language</li> </ul>	<ul style="list-style-type: none"> <li>explains reasoning in clear and coherent mathematical language</li> <li>justifies<sup>5</sup> reasoning using appropriate mathematical language</li> </ul>
	<ul style="list-style-type: none"> <li>shows no evidence of reflection or checking of work</li> <li>can judge the reasonableness of a solution only with assistance</li> </ul>	<ul style="list-style-type: none"> <li>shows little evidence of reflection or checking of work</li> <li>is able to decide whether or not a result is reasonable when prompted to do so</li> </ul>	<ul style="list-style-type: none"> <li>shows some evidence of reflection and checking of work</li> <li>indicates whether the result is reasonable, but not necessarily why</li> </ul>	<ul style="list-style-type: none"> <li>shows ample evidence of reflection and thorough checking of work</li> <li>tells whether or not a result is reasonable, and why</li> </ul>
	<ul style="list-style-type: none"> <li>unable to identify similar<sup>6</sup> problems</li> </ul>	<ul style="list-style-type: none"> <li>unable to identify similar<sup>6</sup> problems</li> </ul>	<ul style="list-style-type: none"> <li>identifies similar<sup>6</sup> problems with prompting</li> </ul>	<ul style="list-style-type: none"> <li>identifies similar<sup>6</sup> problems, and may even do so before solving the problem</li> </ul>
	<ul style="list-style-type: none"> <li>unlikely to identify extensions<sup>7</sup> or applications of the mathematical ideas in the given problem, even with assistance</li> </ul>	<ul style="list-style-type: none"> <li>recognizes extensions<sup>7</sup> or applications with prompting</li> </ul>	<ul style="list-style-type: none"> <li>can suggest at least one extension<sup>7</sup>, variation, or application of the given problem if asked</li> </ul>	<ul style="list-style-type: none"> <li>suggests extensions<sup>7</sup>, variation, or applications of the given problem independently</li> </ul>

## Suggested Assessment Strategies

### Notes on the Rubric

1. For example, diagrams, if used, tend to be inaccurate and/or incorrectly used.
2. For example, diagrams or tables may be produced but not used in the solution.
3. For example, diagrams, if used, will be accurate models of the problem.
4. To *describe* a solution is to tell *what* was done.
5. To *justify* a solution is to tell *why* certain things were done.
6. *Similar* problems are those that have similar structures, mathematically, and hence could be solved using the same techniques.

For example, of the three problems shown below right, the better problem solver will recognize the similarity in structure between Problems 1 and 3. One way to illustrate this is to show how both of these could be modelled with the same diagram:



**Problem 1:** There were 8 people at a party. If each person shook hands once with each other person, how many handshakes would there be? How many handshakes would there be with 12 people? With 50?

**Problem 2:** Luis invited 8 people to his party. He wanted to have 3 cookies for each person present. How many cookies did he need?

**Problem 3:** How many diagonals does a 12-sided polygon have?

Each dot represents one of 12 people and each dotted line represents either a handshake between two people (Problem 1, second question) or a diagonal (Problem 3).

The weaker problem solver is likely to suggest that Problems 1 and 2 are similar since both discuss parties and mention 8 people. In fact, these problems are alike only in the most superficial sense.

7. One type of extension or variation is a “what if...?” problem, such as “What if the question were reversed?”, “What if we had other data?”, “What if we were to show the data on a different type of graph?”.

**Suggested Assessment Strategies****ADAPTING THE RUBRIC**

The problem solving in this unit is spread throughout the activities. That is, not all the components of problem solving as outlined in the rubric are present in each lesson. However, there are examples of each to be found in the series of activities presented.

Examples of these criteria are given below with questions based on a part of one of the activities. This allows you to assess the students' problem-solving abilities in different ways at different times during the unit.

You may wish to share this type of assessment with students. The more aware of the nature of problem solving (as "described" by a rubric) they become, the better problem solvers they will become, and the more willing to try to articulate their solutions and reasons for their choices of various strategies and heuristics.

**ACTIVITY 1, BLM 1**

**UNDERSTANDING:** How well do students understand the use of a model (e.g., Charlie Cube)? How well do they adapt the model to suit their own measurements? Do they have any trouble relating the number of cubes used to their body volume?

For example,

- The "Limited" student may think that the new model must use the same number of cubes as in Charlie Cube.
- The "Acceptable" student will realize that the number of cubes may change, and that the proportions of their measurements and Charlie's will differ also.

**STRATEGIES:** How well do students adapt the cube model to reflect their own measurements?

For example,

- The "Limited" student may have difficulty adapting the model to suit his/her own measurements.
- The "Capable" student will have no difficulty building a new model to suit the student's own measurements.
- The "Proficient" student will be able to answer the questions without building a model or making a drawing.

**ACTIVITY 2, BLM 5**

**STRATEGIES AND REASONING:** What strategies do students use to predict whether or not given objects can fit through a hole in the paper? How well can they justify their choices?

For example,

- The "Limited" student will need to cut the hole before being certain of his/her answers.
- The "Acceptable" student may be unsure of some answers but will be willing to make predictions without cutting.
- The "Capable" student will confidently make predictions and be able to justify them.

## Other Resources

1. *Principles and Standards for School Mathematics*, Jean Carpenter and Sheila Gorg (Ed.), 2000, NCTM  
Includes a general discussion of measurement expectations for each grade level, and how to help students achieve them. The Grade 6-8 section has an intriguing suggestion for an activity dealing with relative sizes, based on Shel Silverstein's poem 'One Inch Tall'.
2. *Measurement, Addenda Series, Grades 5-8*, Grances R. Curcio (Ed.), 1994, NCTM  
This booklet contains problems for grades 5-8 dealing with measurement and estimation. Topics include making, and assessing the validity of, estimates of mass, length, area, volume, capacity, selecting appropriate units of measurement, using a variety of measurement tools, graphing data, and developing formulas based on observations. There is a wide variety of excellent applications to real-world problems.
3. "By the Unit or Square Unit?" *Mathematics Teaching in the Middle School*, B.B. Ferrer, Nov. 2001, pp 132-137  
The article describes teachers' ways of helping students understand perimeter and area and any relationships between them. For example, students were presented with the problem: "For the school carnival, sponsors will pay for advertising their products on signs. How can the school make the most money: by charging the sponsors by the square unit of area, or by measuring the perimeter of the signs and charging by the linear unit?" Both regular and irregular shapes (e.g., pentominoes, students' footprints) are measured, using grid paper and string, bringing out some of the less intuitive aspects of how perimeter relates to area.
4. "Rules of Thumb" and "Rules of Thumb 2", Tom Parker, 1983 and 1987, Houghton Mifflin Company, Boston, ISBN 0-395-34642-8 (pbk.) and 0-395-42955-2 (pbk.)  
These two books contain, between them, 1826 "rules-of-thumb" on almost any topic you can think of (e.g., body measurements, children, distance, travel, and weather). The index helps one locate any one of these topics. The rules-of-thumb are presented as they were submitted and sometimes contradict each other. Parker makes no claims for their validity. One good problem for students would be to select a rule-of-thumb and have students devise an experiment to test its validity.
5. "What Is A "Good Guess", Anyway?", by Frances Kuwahara Lang, pp 462-466, *Teaching Children Mathematics*, April 2001, NCTM  
This theoretical discussion analyzes the strategies children use in estimating and gives examples of how teachers can help them develop and refine their estimation and measurement skills.
6. "Mousemaze Tournament: Connecting Geometry and Measurement" by Shirley Curtis, pp 504-509, *Teaching Children Mathematics*, May 2001, NCTM  
Students are challenged to collaboratively design and construct mouse mazes, based on student-generated criteria, and then participate in a tournament for which they also develop the rules. An open-ended activity which stimulates a high degree of involvement, and appeals to students of varying levels of mathematical experience.

## Other Resources

7. “Using a Lifeline to Give Rational Numbers a Personal Touch” by Wanda Weidemann, Alice Mikovch, and Jane Braddock Hunt, pp 210-215, *Teaching Children Mathematics*, December 2001, NCTM  
A creative activity in which students develop a personal ‘time-ruler’ for their lives. Using one year as the unit of time prompts the need for representing and manipulating non-integer numbers. An assessment rubric and cross-curricular connections are included.
8. “How Big Was the Cat?” by L.E. Sakshaug and K.A. Wohlhuter, pp 350-351, *Teaching Children Mathematics*, February 2001, NCTM  
This ‘problem solver’ poses the problem faced by a boy who sees the ‘biggest’ cat while on a walk in the woods, but has difficulty convincing his family that it really was the ‘biggest’. Students are invited to help him justify his claim, stimulating an exploration of comparative methods for estimating size. Variations are also suggested.
9. “Rubber-Band Rockets”, by M.E. Hynes, J.K. Dixon, and T. Lott Adams, pp 390-393, *Teaching Children Mathematics*, March 2002, NCTM  
In constructing and experimenting with rubber-band rockets, students make and test conjectures, measure angles and distances, explore the material properties of rubber bands, and display and analyze data in a group setting during this cross-curricular activity.
10. “Food for Thought”, by G.C. Poke, N. Ewing, and D. Stevens, pp 148-150, *Teaching Children Mathematics*, November 2001, NCTM  
This ‘Math by the Month’ activity suggests a multitude of ways to use food to prompt mathematical thinking, e.g., (i) collect soft-drink cups and explore their relative sizes; (ii) weigh apples daily in three modes: whole, peel plus apple, and sliced, and chart the changes over one week.
11. “Sizing Up the River”, by Alice H. Merz and Carrie Thomason, *Teaching Children Mathematics*, pp 149-155, November 2002, NCTM  
Students explore river depth and water flow of a nearby river, after studying models in the classroom. The article outlines the teaching sequence and discusses students’ responses. Two BLMs are provided. During the activity students measured length, depth, and time, and calculated speed and volume.
12. “Honoring Traditions: Making Connections with Mathematics through Culture”, *Teaching Children Mathematics* by Deborah McLean, pp 184-188, November 2002, NCTM.  
This article presents a brief history of measurement, moving from the informal to the formal. A specific example from the Yup’ik culture of southwest Alaska involves the design of a hooded shirt using the distance from shoulder to wrist and across the back of the shoulders measured in ‘handwidths’. Uses of such nonstandard units in other contexts and their importance both historically and in the present day are explored, along with the importance of teaching such culturally-based information.

## Other Resources

13. Science Is ..., (2nd ed.), by Susan V. Bosak et al, Scholastic Canada Ltd., 1991

This practical book has an extensive collection of activities/experiments encompassing environmental studies and mathematics, developed with input from students, teachers and parents. While directed at children ages 6-14, these hands-on activities provide fun and ample learning opportunities for all. Permission to copy for classroom use is granted.

14. “The Environmental Math Workbook”, by Evannah Sakamoto, Courseware Solutions Inc., Toronto, 1992.

This book provides activities tied to environmental issues, such as waste collection and analysis. Activities connect mathematics with environmental studies in a “user-friendly” way.

15. “Weather Watch” by Valerie Wyatt, 1990, Kids Can Press, Toronto.

This easy-to-read book gives information about such things as the nature of lightning, types of clouds, and how to measure rain drops or wind direction. Included are simple experiments for students to create and measure various weather phenomena (e.g., ‘Smog In A Jar’, ‘How Wet Can It Get?’, or ‘Thunder In a Pop Can’).