



## Problem of the Month

### Problem 0: September 2021

#### Problem

Some friends are playing a game involving ten cards numbered 1 through 10. In part (a), Adina, Budi, and Dewei are the players. In parts (b) and (c), Adina, Budi, Charlie, and Dewei are the players. To play the game, each player other than Dewei chooses a card and shows it to all other players, but no player looks at their own card. The game consists of a dialogue with the goal being for all players holding a card to deduce the integer on their own card. In each part of this question, the dialogue is given in the order the statements/questions occurred. **No player is allowed ask a question to which they already know the answer.**

- (a) Given the dialogue below, determine the integers on Adina's and Budi's cards.
1. (Adina) Is the integer on my card larger than the integer on Budi's card?
  2. (Dewei) No.
  3. (Budi) I know the integer on my card.
  4. (Adina) I know the integer on my card.
- (b) After the dialogue below, Adina, Budi, and Charlie each know the integer on their own card. Determine all possibilities for the integers on their cards.
1. (Adina) Is the sum of the integers on the cards a perfect square?
  2. (Dewei) Yes.
- (c) Given the dialogue below, determine all possibilities for the integers on the cards.
1. (Adina) Are the integers on any of the cards prime?
  2. (Dewei) No.
  3. (Budi) Is the sum of the integers on the cards prime?
  4. (Dewei) Yes.
  5. The three statements below occur simultaneously.
    - (Adina) I do not know what integer is on my card.
    - (Budi) I do not know what integer is on my card.
    - (Charlie) I know what integer is on my card.
  6. The two statements below occur simultaneously
    - (Adina) I still do not know what integer is on my card.
    - (Budi) I now know what integer is on my card.
  7. (Adina) I now know what integer is on my card.
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## Hint

- (a) Would this dialogue be possible if Adina saw a 3 on Budi's card? Make sure you keep in mind that no player will ever ask a question to which they already know the answer!
  - (b) To narrow the search for the answer, determine all possible sets of three distinct integers between 1 and 10 inclusive that have a sum equal to a perfect square. Going from there, you might want to explore what would happen for some particular configurations of the cards. For instance, if the integers are 1, 2, and 6, would it be possible for all players to determine the integer on their card from the dialogue given?
  - (c) The general strategy for this problem is similar to that in (b), but you will need to carefully examine how the players would be able to eliminate possibilities based on what the other players are able to infer. Experimenting with various configurations of the cards will likely be helpful.
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