



## Problem of the Month

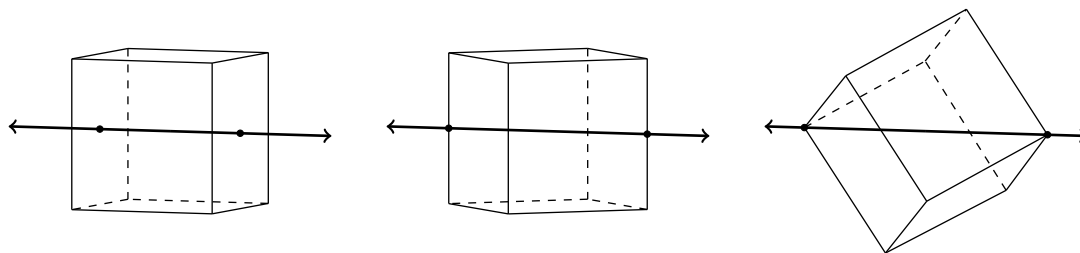
### Problem 8: May 2022

In each part of this problem, a unit cube is positioned with its centre at the origin and is rotated about the  $x$ -axis so that it sweeps out a new “solid of revolution”. To visualize this solid, you might imagine a cube being rotated very quickly about a fixed axis to produce an illusion of the solid. [This is the same phenomenon as when a rotating propellor or fan blade looks like a disk.] For example, if the cube is originally positioned so that the  $x$ -axis passes through the centres of two opposite faces, then the solid of revolution is a cylinder.

The solid of revolution depends on the original position of the cube. In each part, information is given to describe the original position of the cube and the goal is to describe the region in the  $(x, y)$ -plane intersected by the solid of revolution.

- The cube is positioned so that the  $x$ -axis passes through the centres of two opposite faces. As mentioned in the preamble, the solid is a cylinder.
- The cube is positioned so that the  $x$ -axis passes through the midpoints of two opposite edges of the cube (that is, two edges that are parallel and are not edges of the same face).
- The cube is positioned so that the  $x$ -axis passes through two opposite vertices of the cube (that is, two vertices that are not on a common face).

Below, from left to right, are diagrams of the original position of the cube for parts (a), (b), and (c), respectively. In order to avoid clutter in the diagrams, only the  $x$ -axis is included.



Notes:

- In the solutions, regions in the  $(x, y)$ -plane will have descriptions of the form “the region between  $x = a$  and  $x = b$  above the graph of  $y = f(x)$  and below the graph of  $y = g(x)$ . You may have some other way of describing the regions.
  - Solids of revolution are studied in calculus. If you already know some calculus and would like an added challenge, you might like to try to compute the area of the regions you find, or even the volumes of the solids of revolution.
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