# Problem of the Month

## Problem 8: May 2024

## Hint

(b),(c) Once the $m$ integers are placed in the leftmost column of a Griffin Grid, the rest of the integers in the grid are uniquely determined.

The definition of a Griffin Grid can be extended to infinite grids. It is useful to consider the infinite (in one direction) grid with $m$ rows and infinitely many columns numbered 1, 2, 3, and so on indefinitely. In such an infinite Griffin Grid, the first $k$ columns form an $m \times n$ Griffin Grid if and only if the $(n+1)^{\text{st}}$ column consists entirely of 1s.

Instead of filling in the infinite grid with $-1$s and 1s, fill the first column with variables, then start to fill in the grid in general. Keep in mind that the only values that the variables will ever take is $-1$ and 1, so you can assume that every variable squares to 1. For example, if $m = 2$ and the variables are $a$ (in the top cell) and $b$ (in the bottom cell), then both variables in the second column are $ab$, the third column is identical to the first column, and the fourth column has 1 in both cells. Using the observation at the end of the previous paragraph, the number of $m \times n$ Griffin Grids is always equal to the set of solutions to a very specific type of system of equations.

(e),(f) Use the same approach as is outlined for part (c) above. You want to show that there are always lots of columns that give rise to systems of equations that have many solutions and lots that give rise to systems with very few solutions. For example, if the $(n+1)^{\text{st}}$ column (in the general grid with variables) consists entirely of 1s, then there are $2^m$ different $m \times n$ Griffin Grids.