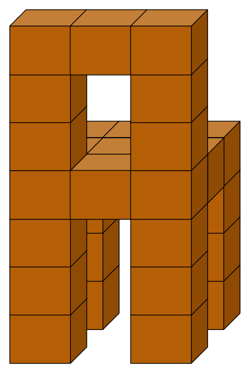
Problem of the Week
Problem A and Solution
Block Furniture

Problem

The Suite Deal Furniture company is famous for their best selling block furniture that is built out of interlocking wooden cubes.

A) Use the diagram below, to determine the number of blocks required to build the chair.



B) Suite Deal wants to make a larger chair, by increasing the leg height by 1 and the seat width and depth by 1. How many blocks are required to build the larger version?

Solution

A) From what we can see in the diagram, the seat of the chair is 3×3 blocks. This is a total of 9 blocks for the seat. Each of the four legs of the chair are 3 blocks tall, so this is a total of $4 \times 3 = 12$ blocks for the legs. We can count the blocks that form the back of the chair and see that there are a total of 7 blocks for the back. This means it takes a total of $9 + 12 + 7 = 28$ blocks to build the chair in the diagram.

B) If the seat width and depth of the chair are increased by 1, then the seat is formed using $4 \times 4 = 16$ blocks. If the leg height of the chair is increased by 1, then each leg would be 4 blocks tall, so you would have $4 \times 4 = 16$ blocks for the legs. The height of the back of the chair is unchanged, but when the seat width increases by 1, you will need 4 blocks across the top of the back. This is one more block than the original for a total of 8 blocks for the back. So, the total number of blocks required is $16 + 16 + 8 = 40$ blocks.

Alternatively we can count the extra blocks we would need to build the bigger chair. We would need to add 1 block to each leg for a total of 4 blocks. We would need to add 7 blocks to the seat, since a 4×4 square has 7 more units than a 3×3 square. And, we would need to add one more block to the back of the chair to handle the new seat width. This means we would need a total of $28 + 4 + 7 + 1 = 40$ blocks for the bigger chair.

Teacher's Notes

This problem illustrates the literal relationship between *linear* and *square* functions. To calculate the square of a number, we could build a square out of blocks. For example, if we want to know the value of 3^2 we can build a square like the seat of the chair. The seat has dimensions 3×3 and there are 9 blocks that form the seat. Thus $3^2 = 9$. Similarly, we can calculate the cube of a number by building a cube out of blocks. For example, the value of 4^3 is equal to the total number of blocks required to build a $4 \times 4 \times 4$ cube. If we count the number of blocks in that cube we would see that we needed 64 in total. Thus $4^3 = 64$.

Another thing we can notice from this problem is that a small change in a linear value leads to a much bigger change in the square value. When we increased the width of the seat from 3 to 4, the number of blocks required to build the square seat increased from 9 to 16. More generally, if we consider any number x that is greater than or equal to 1, and know that another number y is double the size of x, then we can prove that y^2 is always four times as big as x^2. We can check this result with a few examples:

If $x = 5$, then $y = 10$
Therefore, $x^2 = 25$ and $y^2 = 100$
Since $4 \times 25 = 100$ we can say $4 \times x^2 = y^2$

or

If $x = 30$, then $y = 60$
Therefore $x^2 = 900$ and $y^2 = 3600$
Since $4 \times 900 = 3600$ we can say $4 \times x^2 = y^2$

We can prove the general case with algebra:

If $y = 2x$, then $y^2 = (2x)^2$
$(2x)^2 = (2x) \times (2x) = 4x^2$
Therefore, y^2 is always 4 times the size of x^2.