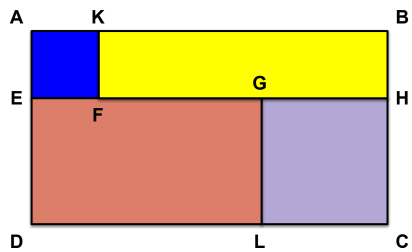
Problem of the Week
Problem A and Solution
Area Issues

Problem

The employees of Sew Inspired need to make a quilt for a special puzzle project. They draw a pattern for the four pieces of cloth they will cut for each section. The pattern includes two square pieces (**AKFE** and **GHCL**), and two rectangular pieces (**KBHF** and **EGLD**).



The area of the piece **AKFE** is 4 square units. The area of the piece **GHCL** is 9 square units. The points **E**, **F**, **G**, and **H** are on the same straight line, and the line segment **FG** is 5 units long.

What is the area of the rectangular section **ABCD**?

Solution

One way to calculate the area of rectangle **ABCD**, is to determine the lengths of its sides. We know the area of the square **AKFE** is 4 square units, and we know that the lengths of the sides of a square must be the same. So if the length of one side of a square is n, then the area of the square must be $n \times n$. By trial and error we can see that $2 \times 2 = 4$, so each side of the square **AKFE** must be 2 units. Similarly, since the area of the square **GHCL** is 9 square units, we can see that $3 \times 3 = 9$. So each side of the square **GHCL** must be 3 units.

Another way to determine the lengths of the sides of square **AKFE**, would be to start with 4 unit squares (using blocks or cut out of paper for example) and determine how to arrange them into a larger square. The only possible arrangement is a 2×2 square.

The opposite sides of a rectangle must be the same length, and the bottom of the square **AKFE** is on the same line as the top of square **GHCL**. This means the length side **AD** is equal to the sum of the lengths of the sides of the two squares. So the length of side **AD** is $2 + 3 = 5$ units. Also, since the length of line segment **FG** is 5 units, the length of line **EH** must be $2 + 5 + 3 = 10$ units. The length of this line is the same as the length of the side **AB** or the side **DC**.

Now we can calculate the area of rectangle **ABCD**. The area of this rectangle is the product of the length of side **AD** and the length of side **AB**. So the area of rectangle **ABCD** is $5 \times 10 = 50$ square units.

Teacher's Notes

Exploring the areas of rectangles or squares is a good way to practice multiplication of positive numbers. The result of calculating $a \times b$ is equivalent to calculating the area of a rectangle with side lengths a and b.

Similarly, exploring the relationship between the area of a square and the lengths of its sides is a good way to learn about *square roots*. However, this exploration only produces half of the answer.

A square root of a number is defined as follows:

$$\text{If } y \text{ is a square root of } x, \text{ then } y \cdot y = x.$$

Since the product of two negative numbers is positive, the value of y could be positive or negative. So the square root of 9 is either 3 or -3. When we use the radical symbol, $\sqrt{\ }$, we are looking for the principal (or positive) square root. Although -3 and 3 are both square roots of 9, the $\sqrt{9} = 3$.

If x is a *perfect square* (i.e. it is the the result of multiplying a rational number by itself), then the result of calculating $\sqrt{x}$ is also a rational number. A *rational number* is any number that can be represented as a fraction $\frac{a}{b}$, where a and b are integers, and b is not 0.

For example, $\sqrt{144}$ is exactly 12. There are also non-integers values that have rational square roots. For example, $\sqrt{\frac{9}{16}}$ is exactly $\frac{3}{4}$.

There are many numbers, however, that are not perfect squares. Square roots of these numbers are known as *irrational numbers*. An irrational number cannot be represented precisely as a fraction. If we try to write an irrational number in decimal form, we end up with a result where the digits after the decimal place never end, and never repeat a pattern. So, we cannot compute the square root of an irrational number on a calculator exactly. The result will appear as a limited number of digits after the decimal place. The calculator rounds the answer to a fixed number of decimal places.