Problem of the Week
Problem A and Solution
School Expansion

Problem

Emmy Noether Public School needs to add a new wing to its main building. The contractor has the plans for the addition, but some measurements were missing. The new rooms are rectangular and are connected as shown.

The good news is that the contractor remembered some extra details.

- When describing the dimensions of a rectangle, the contractor uses *length* to describe the longer side, and *width* to describe the shorter side.

- The width of the classroom is the same as the length of the supply closet.

- The length of the classroom is three times longer than the width of the supply closet.

- The width of the library and the length of the office are the same.

- The width of the office is half the length of the library.

The contractor needs to order tiles for the new rooms.

A) What are the missing dimensions of the classroom and the office?

B) What is the total area needed to be covered by the tiles?

Solution

A) Since the classroom length is three times longer than the supply closet width, its length is $2 \times 3 = 6$ m. Since the office width is half the length of the library, its width is $8 \div 2 = 4$ m. Here is a diagram with all of the dimensions:
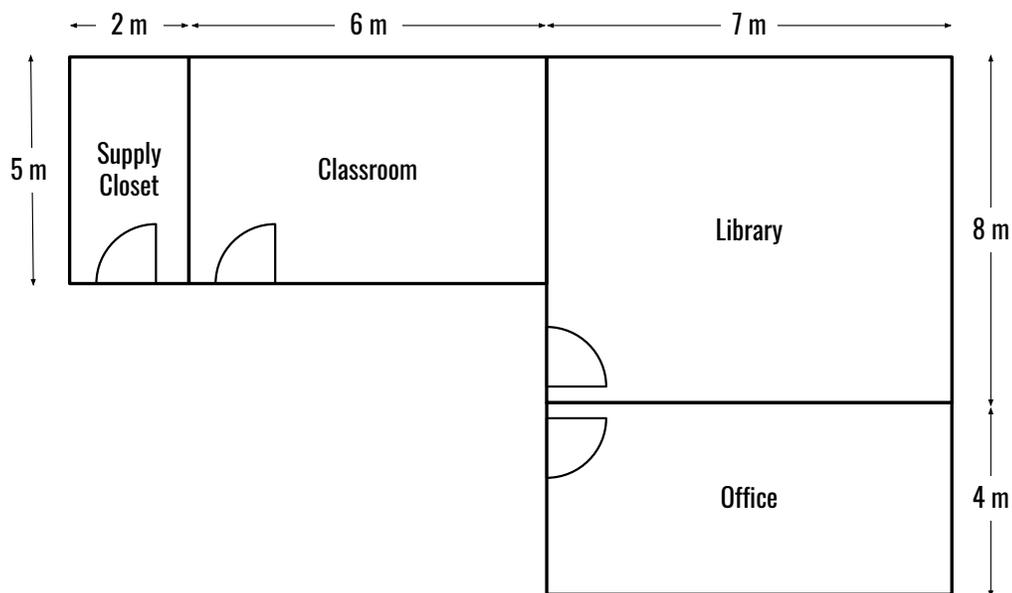
B) There are many ways to calculate the area needed to be covered. One way is to calculate the area of each room and add those numbers together.

- The area of the supply closet is $5 \times 2 = 10$ m^2.
- The area of the classroom is $5 \times 6 = 30$ m^2.
- The area of the library is $7 \times 8 = 56$ m^2.
- The area of the office is $7 \times 4 = 28$ m^2.

So the total area the contractor needs to tile is $10 + 30 + 56 + 28 = 124$ m^2.

Another way to find the total area is to calculate the area classroom based on the area of the supply closet and to calculate the office based on the area of the library. Since the length of the classroom is three times longer than the supply closet width, but the other dimensions are the same, the area of the classroom is three times the area of the supply closet. We can imagine that we could put three closets side-by-side and they would fill in the same area as the classroom. This means that the area of the classroom is $10 \times 3 = 30$ m^2. Similarly we can imagine that the area of the office would fill half of the area of the library. This means that the area of the office is $56 \div 2 = 28$ m^2.

A third way we could calculate the area that needs to be covered in tiles is to square off the diagram, as shown below.
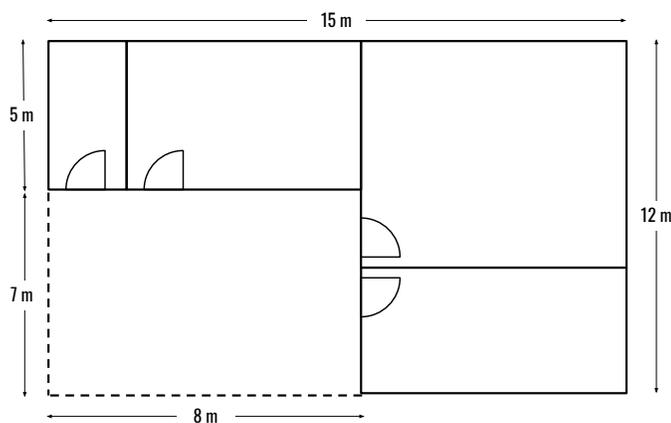
The length across the top is $2 + 6 + 7 = 15$ m. The width along the side is $8 + 4 = 12$ m. The total area of that rectangle is $15 \times 12 = 180$ m^2.

However, there is a section of that rectangle that is not part of the building and hence needs no tiles. That section has a length equal to $2 + 6 = 8$ m and a width equal to $12 - 5 = 7$ m. The section of the diagram with no tile has an area of $8 \times 7 = 56$ m^2.

So the tiled area is the difference between these the areas of these two rectangles: $180 - 56 = 124$ m^2.

Teacher's Notes

Throughout most of history women were not actively encouraged to pursue academia, especially in the fields of mathematics and science. Emmy Noether (the namesake of the school in this problem) was born in Germany in 1882. Her father was a mathematician and she decided to follow in his footsteps. At the time, it was very difficult for a women to study at a university, but Noether was allowed to audit some courses at the University of Göttingen with the permission of the professor. One of the classes she attended was taught by David Hilbert, who was one of the most influential mathematicians of his time. Hilbert and others encouraged Noether in her academic pursuits. After completing her Ph.D., despite objections by some faculty, Neother taught classes and continued her research at the University of Göttingen. However she was only allowed to teach classes that were officially listed under Hilbert's name. Based on the work she did throughout her career, Emmy Noether is recognized as one of the leading mathematicians of the early 20th century.

During the time that she was teaching at the University of Göttingen, Maria Göppert became a student of mathematics and physics. After marrying her husband, Joseph Edward Mayer, she moved to the United States. Like Noether, Göppert-Mayer had restrictions on her earning potential as a researcher since she was a woman. However, she continued her academic pursuits at institutions such as Johns Hopkins, Columbia University and the University of Chicago. Among her other accomplishments, Göppert-Mayer was a member of the Manhattan Project - the group that developed the first nuclear weapons for the United States. Eventually, Göppert-Mayer became the second woman to receive the Nobel Prize in Physics for research she did after World War II.

As of 2019, only three women have received a Nobel Prize in Physics: Marie Currie, Maria Göppert-Mayer, and Donna Strickland. Strickland received her prize in 2018 (more than 50 years after Göppert-Mayer) for research in the field of lasers. In her Ph.D. thesis, Strickland cited the research of Göppert-Mayer's Ph.D. thesis.

Sources:

- https://en.wikipedia.org/
- https://www.britannica.com/