Problem of the Week
Problem A and Solution
Population Approximation

Problem

Ten years ago Anytown had a population of approximately 8000. Jessie knew the digits 3, 7, 8, 9 were part of the actual population at that time, but could not remember the order of the digits. What might the actual population of Anytown have been if each digit is used once?

Justify your answer.

Solution

If we are looking for a number that is close to 8000, the best options would start with either an 8 or a 7. Using the digits $3, 7, 8, 9$ each exactly once we will find the number closest to 8000 but greater, and then find the number closest to 8000 but smaller.

Closest Number to 8000 but greater

For numbers that start with an 8, the smallest value would have the hundreds digit as small as possible, the tens digit would be the next smallest number, and ones digit would be the largest number. This means that the number starting with 8 that is closest to 8000 but greater and uses each of the digits 3, 7, 8, 9 is 8379. This number is almost 400 more than 8000.

Closest Number to 8000 but smaller

For numbers that start with a 7, the largest value would have the hundreds digit as large as possible, the tens digit would be the next largest number and the ones digit would be the smallest number. This means that the number starting with 7 that is closest to 8000 but smaller and uses the digits 3, 7, 8, 9 is 7983. This number is 17 less than 8000.

If we were looking for the number closest to 8000 that uses the digits 3, 7, 8, 9, we would select 7983.

However, we were not asked for the closest number to 8000 but rather a number that could be reasonably approximated by 8000. In one case, we could argue either of 7938 or 7983 since they are both within 100 of 8000.

We could also argue the numbers 7839 or 7893 or 7938 or 7983 or 8379 or 8397 since each is within 400 of 8000 and could be reasonably rounded to 8000.

Each of the above arguments could act as justification for your answer.

Teacher's Notes

The numbers 7983, 7938, and 8379, or even 8397, 7893 and 7839 are all reasonable choices as a population of Anytown, since the result of rounding these numbers to the nearest 1000 gives us 8000 in each case. As quantitative values get bigger, we tend to care less about the precise digits of those values, and are interested mostly in the most *significant digits*.

Calculators are only able to display a limited number of digits. If the calculated values get too big or too small they are usually shown in *scientific notation* or *E-notation*. Numbers written in this format show some number of digits multiplied by a power of 10. For example, the number of atoms in a mole is approximately 6.022×10^{23}, and the mass of an electron is approximately 9.10938×10^{-31} kg. The precision of a number written in scientific notation is determined by the number of significant digits. The mole size has four significant digits and the mass has six significant digits.

In some situations we might not care about precision at all. We may want to have a general sense of the size of some measured value in terms of its *order of magnitude*. In these cases, we are interested in comparing values to the closest power of 10. For example, consider the population of Austria (approximately 8.7 million in 2016) and the population of India (approximately 1.3 billion in 2016). We are unlikely interested in comparing the differences in these numbers by subtracting them. We can say that the order of magnitude difference between the population of India and Austria is 2. This means that the population of India is approximately 10^2 or 100 times that of the population of Austria. However, if we compare the population of Canada (approximately 36 million in 2016) to the population of the United States (approximately 323 million in 2016) we could say that the order of magnitude difference between the population of the United States and Canada is 1. These descriptions give us a broad sense of the relative sizes of the populations.