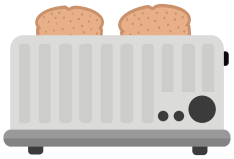


Problem of the Week
Problem A and Solution
Breakfast Food

Problem

For breakfast, Liz always has a drink, some yogurt, and toast. She likes to drink milk, water, or juice. She likes strawberry, blueberry, raspberry, or vanilla yogurt. She likes whole grain or pumpernickel toast. Liz would like a different combination of a drink, yogurt, and toast every day.

A) List all the different breakfast combinations she could have where she does not drink juice.

B) What is the maximum number of days that will pass before she will have to eat and drink exactly the same combination as a previous breakfast?

Solution

A) We can create a table to determine all the possible combinations.

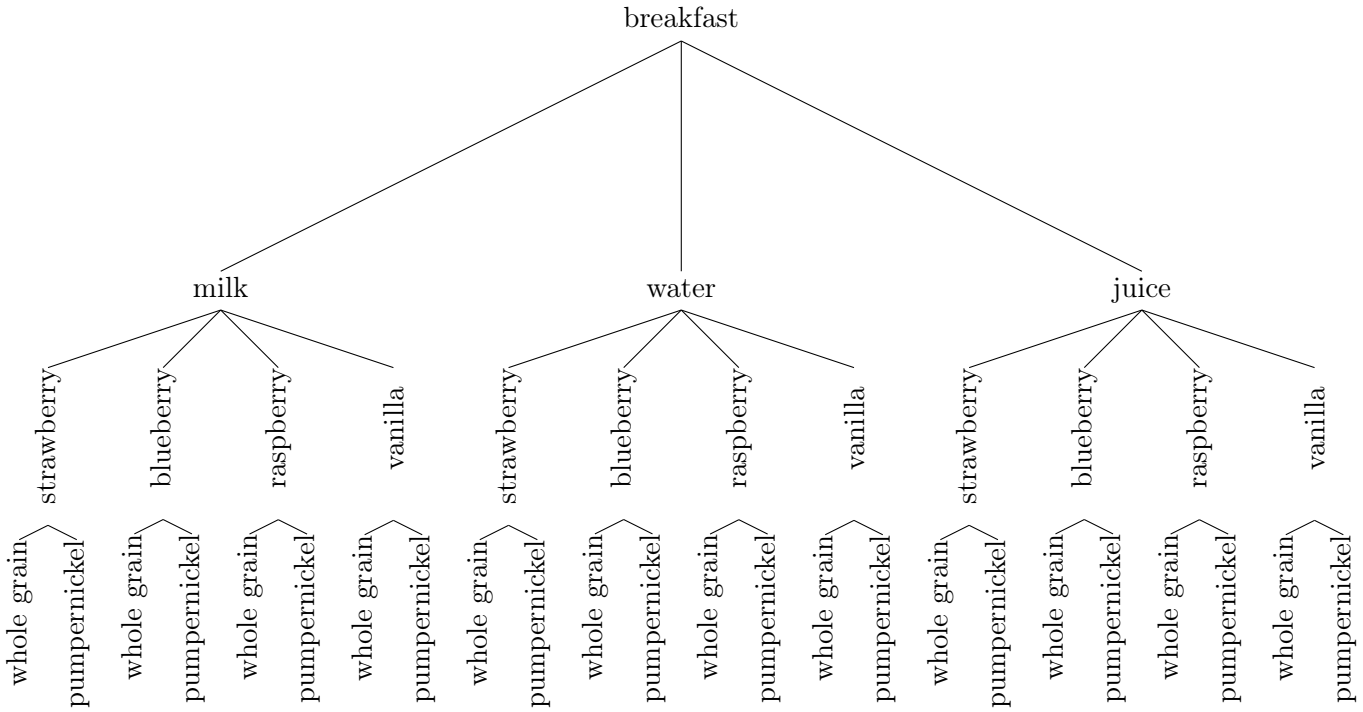
Drink	Yogurt	Toast
milk	strawberry	whole grain
milk	strawberry	pumpernickel
milk	blueberry	whole grain
milk	blueberry	pumpernickel
milk	raspberry	whole grain
milk	raspberry	pumpernickel
milk	vanilla	whole grain
milk	vanilla	pumpernickel
water	strawberry	whole grain
water	strawberry	pumpernickel
water	blueberry	whole grain
water	blueberry	pumpernickel
water	raspberry	whole grain
water	raspberry	pumpernickel
water	vanilla	whole grain
water	vanilla	pumpernickel

Notice the patterns in the table. For example, the eight combinations of yogurt and toast are duplicated, first in combination with milk and then in combination with water.

B) We could extend the table from the previous page to include the juice choice. However, we observe that the eight combinations of yogurt and toast would be repeated for each of the three drink options (milk, water and juice). We then calculate the number of distinct combinations by multiplying: $3 \times 8 = 24$.

We could also look at this calculation by noting that there are 3 choices for a drink. For each of these 3 choices, there are 4 choices of yogurt. So there are $3 \times 4 = 12$ choices of drink and yogurt. And for each of these 12 choices, there are 2 choices of toast. So there are $3 \times 4 \times 2 = 24$ choices of drink, yogurt and toast.

Another way to view the breakfast combinations would be with a tree diagram. This tree shows all of the variations that Liz could have.



Both the calculation and the tree show that there are 24 different breakfast combinations that Liz could have. So at most 24 days could pass before Liz must eat a breakfast combination that she has previously eaten.

Teacher's Notes

The second approach to solving the problem used a *tree* to organize all of the possible breakfast combinations. A tree is a well defined structure used in a field of mathematics known as *graph theory* and is often used in computer science to manage data.

The data in the tree is contained in *nodes* also known as *vertices*. Nodes are connected by *branches* also known as *edges*. This particular tree has a *root*, which is essentially a starting point. In this case, the root is the node that contains the word "breakfast". The nodes at the bottom of the tree are known as *leaf nodes* since they do not have any branches below them. You can follow a *path* from the root node of the tree to any of its nodes. There is only one possible path you can follow to get from the root to any particular node. The paths that we follow from the root of this tree to each of its leaf nodes describe the breakfast combinations that Liz might eat. For example, if you start at the root you could take the following path:

$$\text{breakfast} \rightarrow \text{water} \rightarrow \text{blueberry} \rightarrow \text{pumpernickel}$$

Trees can be used in all sorts of applications. For example, trees can be used to show relationships in a hierarchy or for making logical decisions or for enumerating combinations or for searching efficiently, to name a few possibilities.