Problem of the Week
Problem A and Solution
Caring Community

Problem

The local community centre is organizing a food drive so that they can provide meals for those in need. Three volunteers have offered to gather food items in bulk. Lance collected crates that have 8 cans of tuna in each. Andre has bins that contain 4 bags of pasta in each. Amal gathered cartons with 6 boxes of oatmeal in each.

The team wants to provide food hampers for 24 families. Each hamper will have three cans of tuna, one bag of pasta, and two boxes of oatmeal. How many crates of canned tuna, bins of pasta, and cartons of oatmeal will each volunteer need to bring?

Jusify your answer.

Solution

The first step is to calculate the number of each item we need to supply the 24 families. We need $24 \times 3 = 72$ cans of tuna, $24 \times 1 = 24$ bags of pasta and $24 \times 2 = 48$ boxes of oatmeal.

We can use division to determine how many of the bulk containers we need to satisfy the needs of the food drive.

- Since each crate has 8 cans of tuna, we will need $72 \div 8 = 9$ crates to provide enough tuna.
- Since each bin contains 4 bags of pasta, and each family will be given one bag, we will need $24 \div 4 = 6$ bins of pasta.
- Since each carton contains 6 boxes of oatmeal, we will need $48 \div 6 = 8$ cartons of oatmeal.

Teacher's Notes

The numbers in this problem were carefully chosen. The number of families (24) is a multiple of all of the other numbers in the problem (1, 2, 3, 4, 6, 8). By choosing a number of families that is a multiple of all of the other numbers, we are guaranteed that there will not be any leftovers in the crates, bins, and cartons that we gather.

The number 24 is actually the smallest positive integer that is a multiple of 1, 2, 3, 4, 6, and 8. We call such a number a *least common multiple* or *LCM*. One way to calculate the LCM of a set of numbers it to start by finding the prime factorization of the numbers in the set. In this case, 2 and 3 are prime numbers themselves. The prime factorizations of the rest of the numbers are:

$$4 = 2 \times 2$$
$$6 = 2 \times 3$$
$$8 = 2 \times 2 \times 2$$

Next we identify the maximum number of times that each prime number appears in any single factorization. The only prime numbers that appear in the factorizations of these numbers are 2 and 3. The number 2 appears a maximum of three times, and the number 3 appears only once. To find the LCM, we multiply the prime factors together, assuring that we include maximum number of each factor appearing in each individual factorization. So in this example, we need to multiply $2 \times 2 \times 2 \times 3 = 24$ to find the LCM.

One of the uses of the LCM is finding a common denominator when adding or subtracting fractions.