



## Problem of the Week

### Problem A and Solution

#### Watching Carefully

#### Problem

Steve is a birdwatcher. On one of his walks he saw a total of 28 birds. He saw twice as many chickadees as blue jays. He saw three fewer blue jays than woodpeckers. He saw one bald eagle and 10 chickadees. He also saw some herons.

How many herons, blue jays and woodpeckers did he see?

#### Solution

- Since Steve saw twice as many chickadees as blue jays, he must have seen  $10 \div 2 = 5$  blue jays.
- Since he saw three fewer blue jays than woodpeckers, he must have seen  $5 + 3 = 8$  woodpeckers.
- The total number of chickadees, blue jays, woodpeckers, and bald eagles is  $10 + 5 + 8 + 1 = 24$ .
- Since Steve saw a total of 28 birds, he must have seen  $28 - 24 = 4$  herons.





## Teacher's Notes

This problem could be solved algebraically. We could use  $c$  to represent the number of chickadees,  $h$  to represent the number of herons,  $b$  to represent the number of blue jays,  $w$  to represent the number of woodpeckers, and  $e$  to represent the number of eagles. Then we could write the following equations:

$$2 \times b = c \quad (\text{eqn 1})$$

$$w - 3 = b \quad (\text{eqn 2})$$

$$e = 1 \quad (\text{eqn 3})$$

$$c = 10 \quad (\text{eqn 4})$$

$$c + h + b + w + e = 28 \quad (\text{eqn 5})$$

This is known as a system of equations. To be sure we can solve the system, we must have at least as many *linearly independent* equations as we have variables. Two equations are *linearly dependent* if one is a multiple of the other. In this case, we have 5 linearly independent equations with 5 variables. We are guaranteed to have enough information to find the values of the unknown variables. Now we can solve the system. We already know the values for  $c$  and  $e$ .

We can substitute  $c = 10$  into (eqn 1) to get  $2 \times b = 10$  so  $b = 5$ .

Now we can substitute  $b = 5$  into (eqn 2) to get  $w - 3 = 5$  so  $w = 8$ .

Now we can substitute the values for  $c$ ,  $b$ ,  $w$ , and  $e$  into (eqn 5) to get

$$10 + h + 5 + 8 + 1 = 28$$

$$h + 24 = 28$$

$$h = 4$$

