Problem of the Week
Problem A and Solution
Managing Muffins

Problem

The family studies class baked two dozen muffins. The class plans to share one third of the muffins with the school office staff, and one quarter of the muffins with Mr. Ahmed's chess club.

A) Which group will receive more muffins? Justify your answer.

B) How many muffins will be left over after they give the office staff and chess club their shares?

Solution

One way to solve this problem is to create a diagram showing the batch of two dozen muffins. Then we can group the diagram into four equal parts (quarters) using solid black lines and three equal parts (thirds) using different background shading. We can arrange the 24 muffins this way:

A) From the diagram, we can see that one quarter of the batch is equal to 6 muffins, and that one third of the batch is equal to 8 muffins. Since the school office staff received one third of the muffins, they will get more muffins.

B) The class started with $2 \times 12 = 24$ muffins. They gave away a total of $6 + 8 = 14$ muffins. That means they will have a total of $24 - 14 = 10$ muffins left over.

Teacher's Notes

Consider the relationship between a positive integer n and the fraction $\frac{1}{n}$. We can think of $\frac{1}{n}$ as being one part of a whole that has n parts in total. So the fewer parts we have, the bigger each individual part must be. This means that if we compare two positive fractions, $\frac{1}{a}$ and $\frac{1}{b}$, if $a < b$ we know that $\frac{1}{a} > \frac{1}{b}$. For example, since $3 < 4$ we know that $\frac{1}{3} > \frac{1}{4}$.

As n increases in size, then $\frac{1}{n}$ decreases in size proportionally. In other words, if n is a very large number, then $\frac{1}{n}$ is a very small number. Since there are an infinite number of integers, there are also an infinite number of fractions in the form $\frac{1}{n}$. As n grows bigger the value of $\frac{1}{n}$ gets closer to 0, although it never actually reaches 0. We can say that the limit of $\frac{1}{n}$, as n approaches infinity, is 0. This can be written mathematically as:

$$\lim_{n \to \infty} \frac{1}{n} = 0$$

The study of limits is a fundamental part of calculus.