Problem of the Week
Problem A and Solution
Fall Fair Fare

Problem

Avneet is at the Fall Fair with her friends. They plan to go on 10 rides each. They are trying to figure out which is the best price to pay for admission. The fair has two options:

Option #1
  Admission: \$15
  Single ride ticket: \$3
Option #2
  Admission including unlimited ride wristband: \$35

A) Which option should Avneet choose? Justify your answer.

B) What is the most number of rides Avneet could plan to go on that would make Option #1 the better choice? Justify your answer.

Solution

A) If Avneet chooses Option #1 and she goes on 10 rides, the ride tickets will cost $10 \times 3 = \$30$. Then the total cost for the fair with this option would be $15 + 30 = \$45$. Since Option #2 costs \$35, this would be the better choice.

B) One way to figure out this would be to make a table showing the number of rides and the cost of Option #1.

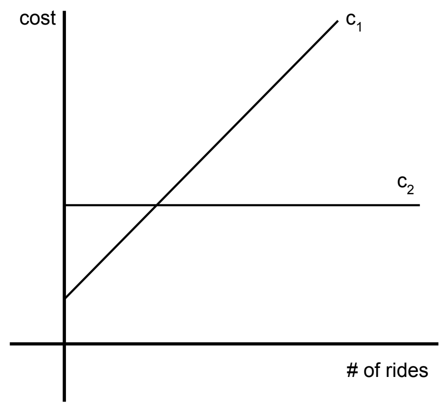
# of Rides	Total Cost (\$)
0	15
1	18
2	21
3	24
4	27
5	30
6	33
7	36

So the most number of rides you can take to make Option #1 the better choice would be 6. Alternatively, since we must pay \$15 for Option #1 no matter how many rides we take, and we must pay \$35 for Option #2 no matter how many rides we take, then the difference between these amounts describes how much of the admission price for Option #2 is covering the cost of the individual rides. This difference is $35 - 15 = \$20$. If we had \$20 for rides that cost \$3 each, we can figure out how many rides we can take by using division: $20 \div 3 = 6$ remainder 2. Since there would be \$2 unspent with Option #2 when we take 6 rides, then Option #1 would be a better choice.

Teacher's Notes

In this problem, we could describe these two options as mathematical functions.
Option #1 could be written as $c_1 = 3 \cdot r + 15$, where r is the number of rides a person takes.
Option #2 could be written as $c_2 = 35$.

If can describe options using mathematical functions, then we have the ability to analyze and compare those functions. In this case, the function describing Option #1 is a **linear** function. This means that the value of c_1 grows at approximately the same rate as the number of rides that Avneet takes. The function describing Option #2 is a **constant** function. This means that the value of c_2 is unaffected by the number of rides that Avneet takes. We can draw a graph showing the difference:



At some point, the linear function crosses the constant function. After that point, the cost of the linear function is always greater than the cost of the constant function.

In computer science, we often use broad categories to compare different functions. Generally, we expect a constant function to be more efficient than a linear function, although, as we see in this problem, a constant function is not necessarily better for **all** values. However, since we normally care about efficiency when dealing with large values, we usually ignore the small cases where the linear function performs better.