Problem of the Week
Problem A and Solution
Patterned Savings

Problem

Rebecca wants to save money starting on January 1. She decided to collect money in a jar in the following way. Every day she puts a quarter in the jar. Every third day, starting on January 3, she puts a loonie in the jar. Every fifth day, starting on January 5, she puts a toonie in the jar. So some days she puts one coin in the jar, some days she puts two coins in the jar, and some days she puts three coins in the jar.

In Canada, a quarter is worth 25 cents, a loonie is worth one dollar, and a toonie is worth two dollars. There are 100 cents in one dollar.

A) How many coins does she put in the jar on January 12?

B) How many coins does she put in the jar on January 26?

C) How many coins does she put in the jar on January 30?

D) How many coins in total does she have in the jar by the end of January?

E) If she keeps saving this way throughout the year, how much money will she have after 90 days? Try to figure this out without counting the money she puts in the jar every day for all 90 days.

Solution

One way to answer most of these questions is to make a table that keeps track of how many coins are deposited each day and how much money is accumulated each day.

Day	Quarters Added	Loonies Added	Toonies Added	Money Added ($)	Total Coins	Total Money Money ($)
1	1	0	0	0.25	1	0.25
2	1	0	0	0.25	2	0.50
3	1	1	0	1.25	4	1.75
4	1	0	0	0.25	5	2.00
5	1	0	1	2.25	7	4.25
6	1	1	0	1.25	9	5.50
7	1	0	0	0.25	10	5.75
8	1	0	0	0.25	11	6.00
9	1	1	0	1.25	13	7.25
10	1	0	1	2.25	15	9.50
11	1	0	0	0.25	16	9.75
12	1	1	0	1.25	18	11.00
13	1	0	0	0.25	19	11.25
14	1	0	0	0.25	20	11.50
15	1	1	1	3.25	23	14.75

We could continue the table, and we would see that the amounts of coins and money added would be repeated exactly every 15 days. From this table, we observe that on every day that is a multiple of 3 (except on day 15) Rebecca adds two coins to the jar, and on every day that is a multiple of five (except on day 15) Rebecca adds two coins to the jar. On day 15, which is a multiple of both 3 and 5, Rebecca adds three coins to the jar. On all other days, Rebecca adds just one coin to the jar.

A) From the table, we see that, on January 12, Rebecca added 2 coins to the jar.

B) Since 26 is neither a multiple of 3 nor a multiple of 5, Rebecca will only add 1 coin to the jar.

C) Since 30 is a multiple of both 3 and 5, Rebecca will add 3 coins to the jar.

D) From the table, we see that after 15 days, Rebecca will have saved 23 coins. This pattern will be repeated in the next 15 days. On January 31, she will add one more quarter. So, by January 31 Rebecca will have $23 + 23 + 1 = 47$ coins in the jar.

E) Every 15 days Rebecca will have saved a total of \$14.75. This pattern will be repeated 6 times over a 90 day period. So after 90 days Rebecca will have $14.75 + 14.75 + 14.75 + 14.75 + 14.75 + 14.75 = \$88.50$. Alternatively we could calculate the savings as $6 \times 14.75 = \$88.50$.

Teacher's Notes

This problem shows a repeating pattern every 15 days. In mathematics, we could refer to this kind of repetition as a *periodic function*. The length of the interval between repeated elements is known as the *period* of the function. In this case, the period of the savings function is 15, which is the *least common multiple* or *LCM* of the values 1, 3, and 5. The LCM of a set of numbers is the smallest number that is a multiple of each element of the set. In this case, we are looking for the smallest multiple of each of the individual periods of savings.

Periodic functions appear in mathematics and in the real world. Trigonometric functions such as *sin*, *cos*, and *tan* are periodic functions. Sound waves, phases of the moon, and your blood pressure, are all examples of periodic functions in nature.