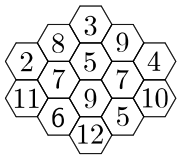


Problem of the Week
Problem B and Solution
Don't Get Vexed by the Wrong Vertex!

Problem

A standard six-sided die has its faces marked with the numbers 1, 2, 3, 4, 5, and 6. The die is fair and each number is used exactly once. A game board is made up of fourteen hexagons, as shown in the diagram. The numbers on the hexagons are arranged randomly. You may place your game piece on any vertex shared by three hexagons. Two standard six-sided dice are then rolled and the two top numbers are added together. If this sum is equal to the number on any of the three hexagons sharing the vertex where your game piece is placed, you win the roll. Which vertices give the best chances to win the roll? Explain your reasoning.

Solution

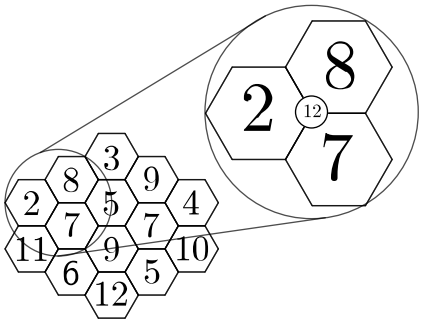
You want to place your game piece at a vertex where the adjacent numbers have the highest probability of being the sums on a roll.

Here are the number of ways each possible sum could occur on a double roll.

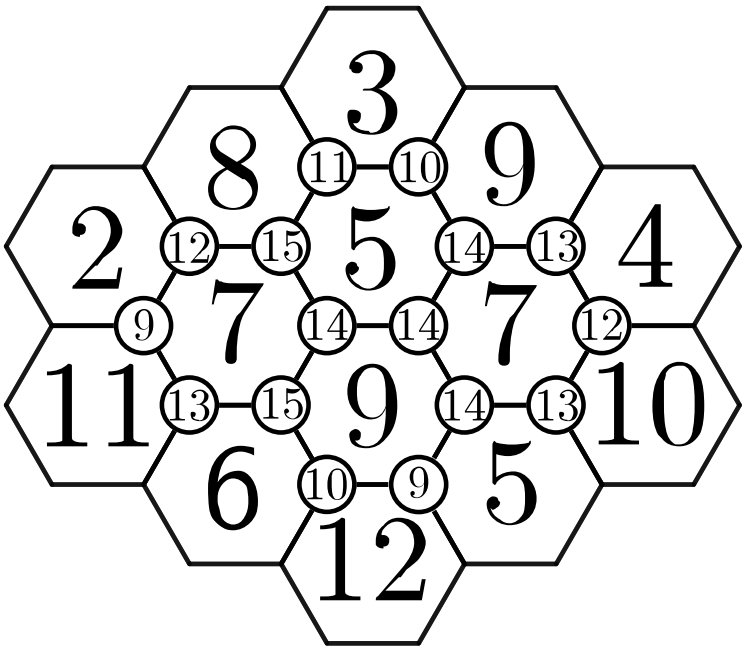
Table of Roll Sums

First Die \ Second Die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

We will need to look at each possible vertex and count the number of times that each of the numbers on the three adjacent hexagons occur in the table above. First, consider when we look at the vertex adjacent to the hexagons with the numbers 2, 8 and 7. The 2 occurs once in the table, the 8 occurs 5 times and the 7 occurs 6 times. Therefore, 2, 8 or 7 occur $1 + 5 + 6 = 12$ times. We will place the 12 on the vertex as shown in the diagram.



We do this for each of the possible vertices and record the numbers on each vertex. The results are shown below.



The number on each vertex is the number of possible rolls out of 36. Therefore, the larger the number the greater the probability of winning the roll. The largest number on any vertex is 15, which occurs on the vertex adjacent to the hexagons with numbers $5, 7, 8$ and on the vertex adjacent to the hexagons with numbers $6, 7, 9$. Therefore, each of these two vertices has a probability of winning of $\frac{15}{36}$.

Thus, in order to have the best chance to win, you want to place your game piece on the vertex that connects the hexagons with numbers $5, 7, 8$, or on the vertex that connects the hexagons with numbers $6, 7, 9$.