



Problem of the Week

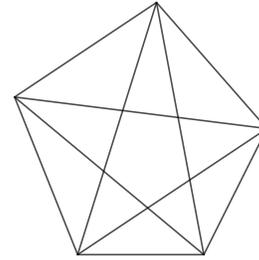
Problem B and Solution

There Are More Than Ten...

Problem

The pentagon shown contains many triangles. Suppose you were asked to count all the triangles in this pentagon.

- a) Think about how you would do this, and write down what your strategy would be.
- b) Now share your strategy with a classmate. Work together using one of your proposed strategies to count all the triangles you can find. Then try the other person's strategy. Did you get the same result?
- c) Compare answers with all your classmates.

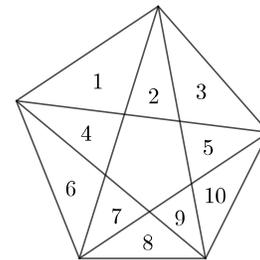


Solution

There are several possible strategies for counting all the triangles in the given pentagonal figure. We have chosen to look at four possible cases.

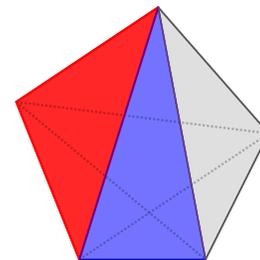
- 1: Count the smallest triangles.

Each triangle has either a side or a vertex on the outer pentagon. There are 10 such triangles.



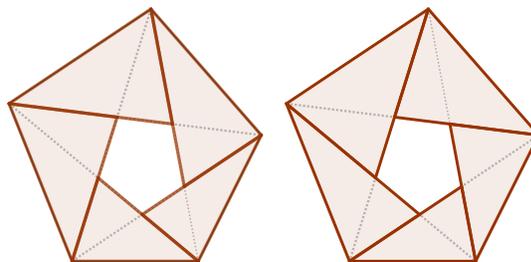
- 2: Count the triangles that have their three vertices on the pentagon.

Since the pentagon has five vertices, there are five sets of three triangles. One such set of three triangles is shown to the right. You might think that there are $5 \times 3 = 15$ triangles but five of the triangles would be counted twice each. (Draw the individual diagrams to verify this for yourself.) So, there are 10 triangles in this case. It is important not to count the same triangles more than once.



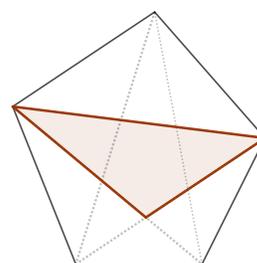
3: Count the triangles that have one side on the outer pentagon that contain two of the smallest triangles that were counted in the first case.

There are two ways to do these combinations, each giving five triangles. So there are $2 \times 5 = 10$ such triangles. These are illustrated in diagrams to the right.



4: Count the interior triangles which have a diagonal as the longest side and a point inside the pentagon as the third vertex.

Since there are five diagonals, there are five such triangles. One of these triangles is shown to the right.



Thus, in total there are $10 + 10 + 10 + 5 = 35$ triangles in total in the given figure.

