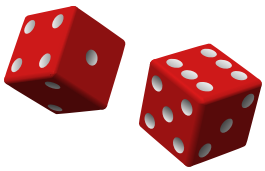

Problem of the Week
Problem B and Solution
These are Sum Products

Problem

Ahmed has two number cubes which he rolls ten times. Each of the 10 rolls gives a **different** combination of the two numbers on the top face (Note, for example, that 5 on the first cube and 1 on the second cube is the same combination as 1 on the first cube and 5 on the second cube). Ahmed finds the product of each pair of numbers and then sums the ten resulting products.

a) Complete the chart of products.
b) What is the set of ten rolls with the lowest possible sum of the products?
c) What is the set of ten rolls with the highest possible sum of the products?
d) Determine 2 different possible sets of ten rolls such that the sum of products is equal to 100. The first set must include every possible roll involving a 3 on at least one of the cubes. The second set must not include any roll with a 3 on either cube.

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Solution

a) The completed chart is shown in the problem statement above.

b) The set of ten rolls would be $1 \times 1 = 1$, $1 \times 2 = 2$, $1 \times 3 = 3$, $1 \times 4 = 4$, $2 \times 2 = 4$, $1 \times 5 = 5$, $1 \times 6 = 6$, $2 \times 3 = 6$, $2 \times 4 = 8$, $3 \times 3 = 9$.
Their sum is $1 + 2 + 3 + 4 + 4 + 5 + 6 + 6 + 8 + 9 = 48$.
These are the ten lowest products in the completed table.

c) The set of ten rolls would be $6 \times 6 = 36$, $5 \times 6 = 30$, $5 \times 5 = 25$, $4 \times 6 = 24$, $4 \times 5 = 20$, $3 \times 6 = 18$, $4 \times 4 = 16$, $3 \times 5 = 15$, $3 \times 4 = 12$, $2 \times 6 = 12$.
Their sum is $36 + 30 + 25 + 24 + 20 + 18 + 16 + 15 + 12 + 12 = 208$.
These are the ten highest products in the completed table.

d) Answers will vary.

One possible set of ten rolls which includes every possible roll involving a 3 on at least one of the cubes is $1 \times 3 = 3$, $2 \times 3 = 6$, $3 \times 3 = 9$, $3 \times 4 = 12$, $3 \times 5 = 15$, $3 \times 6 = 18$, $1 \times 1 = 1$, $1 \times 2 = 2$, $2 \times 5 = 10$, $4 \times 6 = 24$.
In this set, the first 6 rolls in the list must be included and the sum of the products of these rolls is 63. The remaining 4 rolls must add to $100 - 63 = 37$.

One possible set of ten rolls which does not include any roll containing a 3 is
$1 \times 1 = 1$, $1 \times 2 = 2$, $1 \times 4 = 4$, $1 \times 6 = 6$,
$2 \times 2 = 4$, $2 \times 4 = 8$, $2 \times 5 = 10$, $4 \times 4 = 16$, $4 \times 6 = 24$, $5 \times 5 = 25$.

There are many combinations that work.

For Further Investigation: Try putting restrictions on the problem to create different sets of 10 rolls for which the sum of the products is 100.