



Problem of the Week Problem B Powerful Patterns

Consider the first few powers of 2:

$$2, 2 \times 2 = 4, 2 \times 2 \times 2 = 8, 2 \times 2 \times 2 \times 2 = 16, \text{ etcetera.}$$

Powers are often written in a more efficient way, with a superscript which tells how many multiples of 2. For example, 2^3 means $2 \times 2 \times 2$, while 2^5 means $2 \times 2 \times 2 \times 2 \times 2$. The first nine powers of 2 are

$$2, 4, 8, 16, 32, 64, 128, 256, 512$$

Observe that there is a distinct pattern in the ones digits of these powers, namely they follow a repeating sequence

$$\{2, 4, 8, 6, 2, 4, 8, 6, \dots\}$$

Thus, we can predict the ones digit for other powers of 2. For example, the ones digit of 2^{14} is the 14th number in this sequence, which is 4.

- a) Complete the left table below by finding the first five powers of each of the numbers from 1 to 10. What patterns do you observe in the ones digits of these powers?
- b) Complete the right table by finding the first five powers of each of the numbers from 11 to 14. Are there new patterns in the ones digits?
- c) Try to explain why the ones digits of powers of 4 and of powers of 9 follow a slightly different pattern than those for 2, 3, 7, and 8. What do you notice about the patterns?

1	1	1	1	1	1
2	2	4	8	16	32
3					
4					
5					
6					
7					
8					
9					
10					

11					
12					
13					
14					

STRAND PATTERNING AND ALGEBRA

