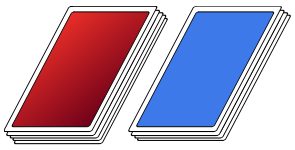

Problem of the Week
Problem C and Solution
Try Your Luck

Problem

At Fall Fairs you often find games of chance. The Red - Blue Card Game is one such game. The following information is known about the game:

- There are five red cards. Each red card has one of the numbers $1, 3, 5, 9$ or 12 printed on it. Each number is used exactly once.
- There are four blue cards. Each blue card has one of the numbers $1, 3, 4$ or 5 printed on it. Each number is used exactly once.
- You draw one red card and one blue card.

If the product of your two numbers is a perfect square, you have drawn a winning pair. What is the probability of drawing a winning pair?

Solution

In order to determine the probability, we must determine the number of ways to obtain a perfect square and divide it by the total number of possible selections of one red card and one blue card. We will compile this information in a table.

Number on Blue Card \ Number on Red Card	1	3	5	9	12
1	1	3	5	9	12
3	3	9	15	27	36
4	4	12	20	36	48
5	5	15	25	45	60

The numbers in the top row of the table, namely, 1, 3, 5, 9, and 12, correspond to the possible values on the red card. The numbers in the first column of the table, namely, 1, 3, 4, and 5, correspond to the possible values on the blue card. The other numbers in the table correspond to the product of the number on the blue card and the number on the red card.

From the table, we see that 20 products are formed. The number 1 is a perfect square, (1×1), and it occurs one time. The number 4 is a perfect square, (2×2), and it occurs one time. The number 9 is a perfect square, (3×3), and it occurs two times. The number 25 is a perfect square, (5×5), and it occurs one time. The number 36 is a perfect square, (6×6), and it occurs two times. Seven of the products are perfect squares. Therefore, the probability of drawing a winning pair is $\frac{7}{20}$.

A game is considered *fair* if the probability of winning is 50%. In this game, the winning probability, as a percentage, is 35%. Can you modify the game so that it is fair?