

Problem of the Week

Problem C

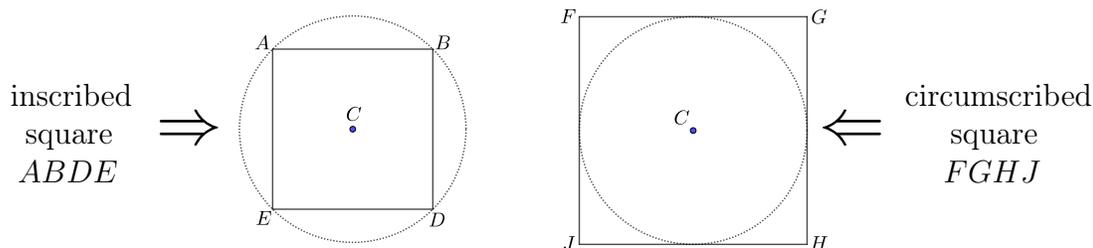
Pi Day Squares

Pi Day is an annual celebration of the mathematical constant π . Pi Day is observed on March 14, since 3, 1, and 4 are the first three significant digits of π .

Archimedes determined lower bounds (minimum values) for π by finding the perimeters of regular polygons inscribed in a circle with diameter of length 1. (An inscribed polygon of a circle has all of its vertices on the circle.)

He also determined upper bounds (maximum values) for π by finding the perimeters of regular polygons circumscribed in a circle with diameter of length 1. (A circumscribed polygon of a circle has all sides tangent to the circle. That is, each side of the polygon touches the circle in one spot.)

We will determine a lower and an upper bound for π by looking at squares inscribed and circumscribed in a circle with centre C and diameter 1.

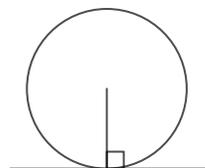
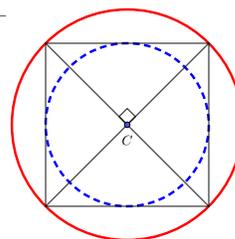


Since the circle has diameter 1, it has circumference equal to π . The perimeter of the inscribed square $ABDE$ will give a lower bound for π and the perimeter of the circumscribed square $FGHJ$ will give an upper bound for π .

Using these squares, determine a lower bound for π (minimum value) and an upper bound (maximum value) for π .

You may want to use the following facts:

1. The diagonals of both an inscribed and circumscribed square meet at the centre of the circle, C , and the diagonals of the square meet at 90° .
2. The *Pythagorean Theorem* states that in a right triangle, the square of the length of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the lengths of the other two sides.
3. The radius of a circle is perpendicular to a tangent of the circle at the point of tangency.



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