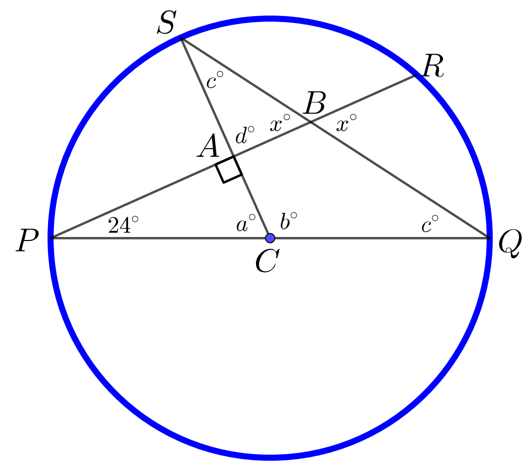
Problem of the Week
Problem C and Solution
Angle Chasing



Problem

A circle with centre C is drawn around $\triangle CQS$ so that Q and S lie on the circumference of the circle. QC is extended to P on the circle. Chord PR intersects CS and QS at A and B, respectively. If $\angle QPR = 24^\circ$ and $\angle CAP = 90^\circ$, determine the measure of $\angle QBR$. ($\angle QBR$ is marked x° on the diagram.)

Solution

Solution 1

The angles in a triangle add to 180°. So in $\triangle CAP$,

$$\angle ACP = 180^\circ - 90^\circ - 24^\circ = 66^\circ = a^\circ.$$

QCP is a diameter and is therefore a straight line. Two angles on a straight line add to 180°, so

$$\angle QCS = 180^\circ - a^\circ = 180^\circ - 66^\circ = 114^\circ = b^\circ.$$

C is the centre of the circle with Q and S on the circumference. Therefore, CS and CQ are radii of the circle and $CS = CQ$. It follows that $\triangle CSQ$ is isosceles and $\angle CSQ = \angle CQS = c^\circ$.

Then in $\triangle CSQ$,

$$\begin{aligned} c^\circ + c^\circ + b^\circ &= 180^\circ \\ 2c + 114 &= 180 \\ 2c &= 66 \\ c &= 33 \end{aligned}$$

Opposite angles are equal, so it follows that $\angle SBA = \angle RBQ = x^\circ$ and $d^\circ = \angle SAB = \angle CAP = 90^\circ$.

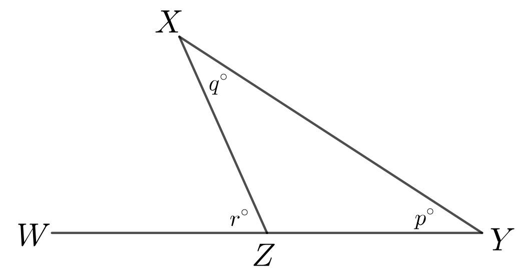
In $\triangle ABS$,

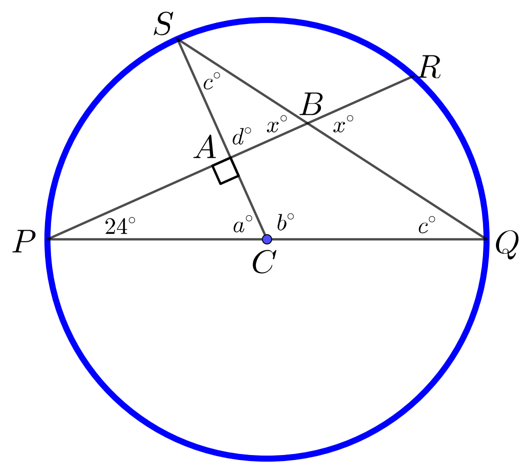
$$\begin{aligned} x^\circ + c^\circ + d^\circ &= 180^\circ \\ x + 33 + 90 &= 180 \\ x + 123 &= 180 \\ x &= 57 \end{aligned}$$

$\therefore \angle QBR = 57^\circ$.

Solution 2

In a triangle, the angle formed at a vertex between the extension of a side and an adjacent side is called an exterior angle. In the top diagram to the right, $\angle XZW$ is exterior to $\triangle XYZ$. The exterior angle theorem states: "the exterior angle of a triangle equals the sum of the two opposite interior angles." In the diagram, $r^\circ = p^\circ + q^\circ$. We will use this result and two of the pieces of information we found in Solution 1.





The angles in a triangle sum to 180°. So in $\triangle CAP$,

$$\angle ACP = 180^\circ - 90^\circ - 24^\circ = 66^\circ = a^\circ.$$

C is the centre of the circle with Q and S on the circumference. Therefore, CS and CQ are radii of the circle and $CS = CQ$. It follows that $\triangle CSQ$ is isosceles and $\angle CSQ = \angle CQS = c^\circ$.

$\angle ACP$ is exterior to $\triangle CSQ$.

$$\begin{aligned} \therefore \angle ACP &= \angle CSQ + \angle CQS \\ a^\circ &= c^\circ + c^\circ \\ 66 &= 2c \\ 33 &= c \end{aligned}$$

$\angle QBR$ is exterior to $\triangle BQP$.

$$\begin{aligned} \therefore \angle QBR &= \angle BPQ + \angle BQP \\ x^\circ &= 24^\circ + c^\circ \\ x &= 24 + 33 \\ x &= 57 \end{aligned}$$

$\therefore \angle QBR = 57^\circ$.