



Problem of the Week

Problem C and Solution

Top Triangle

Problem

The area of $\triangle ACD$ is twice the area of square $BCDE$. AC and AD meet BE at K and L respectively such that $KL = 6$ cm.

If the side length of the square is 8 cm, determine the area of the top triangle, $\triangle AKL$.

Solution

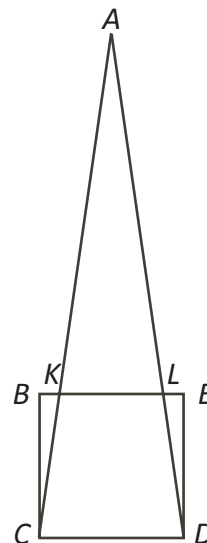
Solution 1

In the first solution we will find the area of square $BCDE$, the area of $\triangle ACD$, the area of trapezoid $KCDL$, and then the area of $\triangle AKL$.

To find the area of a square, multiply the length times the width. To find the area of a trapezoid, multiply the sum of the lengths of the two parallel sides by the height and divide the product by 2.

$$\begin{aligned} \text{Area of square } BCDE &= 8 \times 8 \\ &= 64 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area } \triangle ACD &= 2 \times \text{Area of Square } BCDE \\ &= 2 \times 64 \\ &= 128 \text{ cm}^2 \end{aligned}$$



In trapezoid $KCDL$, the two parallel sides are KL and CD , and the height is the width of square $BCDE$, namely BC .

$$\begin{aligned} \text{Area of trapezoid } KCDL &= (KL + CD) \times BC \div 2 \\ &= (6 + 8) \times 8 \div 2 \\ &= 14 \times 8 \div 2 \\ &= 56 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area } \triangle AKL &= \text{Area } \triangle ACD - \text{Area of trapezoid } KCDL \\ &= 128 - 56 \\ &= 72 \text{ cm}^2 \end{aligned}$$

Therefore, the area of $\triangle AKL$ is 72 cm^2 .

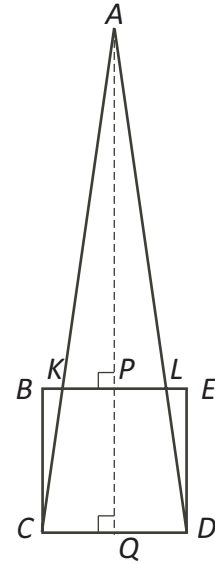




Solution 2

Construct the altitude of $\triangle ACD$ intersecting BE at P and CD at Q . In this solution we will find the height of $\triangle AKL$ and then use the formula for the area of a triangle to find the required area.

To find the area of a square, multiply the length times the width. To find the area of a triangle, multiply the length of the base times the height and divide the product by 2.



$$\begin{aligned}\text{Area of square } BCDE &= 8 \times 8 \\ &= 64 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area } \triangle ACD &= 2 \times \text{Area of Square } BCDE \\ &= 2 \times 64 \\ &= 128 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{But Area } \triangle ACD &= CD \times AQ \div 2 \\ 128 &= 8 \times AQ \div 2 \\ 128 &= 4 \times AQ \\ \therefore AQ &= 32 \text{ cm}\end{aligned}$$

We know that $AQ = AP + PQ$, $AQ = 32$ cm and $PQ = 8$ cm, the side length of the square. It follows that $AP = AQ - PQ = 32 - 8 = 24$ cm.

$$\begin{aligned}\therefore \text{Area } \triangle AKL &= KL \times AP \div 2 \\ &= 6 \times 24 \div 2 \\ &= 72 \text{ cm}^2\end{aligned}$$

Therefore, the area of $\triangle AKL$ is 72 cm^2 .

