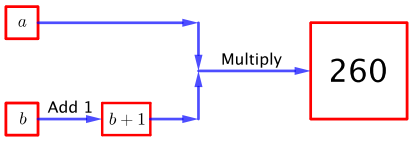
Problem of the Week
Problem C and Solution
A Process - Add then Multiply



Problem

Two positive integers, a and b, are both greater than 1. The second integer, b, is larger than the first integer, a. After b is increased by 1, it is multiplied by a. The resulting product is 260. This information can be represented by the equation $a \times (b + 1) = 260$.

Determine all possible values of a and b that satisfy the equation.

Solution

In considering only the equation $a \times (b + 1) = 260$, we see that we are looking for two integers, each greater than 1, that multiply to 260. The first integer is a and the second integer is an integer that is 1 larger than b. We also want the second integer to be greater than the first integer.

We can generate the following list of valid pairs of integers that multiply to 260:

$$260 = 2 \times 130 = 4 \times 65 = 5 \times 52 = 10 \times 26 = 13 \times 20.$$

There are other pairs of positive integers that multiply to 260 but each of them can be excluded because of the given restrictions.

We can exclude $260 = 1 \times 260$ and $260 = 260 \times 1$ because both integers must be greater than 1.

We can also exclude $260 = 130 \times 2$, $260 = 65 \times 4$, $260 = 52 \times 5$, $260 = 26 \times 10$, and $260 = 20 \times 13$ because the second integer must be larger than the first integer.

Since b is 1 less than the second integer in each valid product, we can generate the pairs of integers that satisfy the equation $a \times (b + 1) = 260$. We will write the integer pairs as ordered pairs in the form (a, b).

The first ordered pair is $(2, 129)$ since $2 \times (129 + 1) = 2 \times 130 = 260$.
The second ordered pair is $(4, 64)$ since $4 \times (64 + 1) = 4 \times 65 = 260$.
The third ordered pair is $(5, 51)$ since $5 \times (51 + 1) = 5 \times 52 = 260$.
The fourth ordered pair is $(10, 25)$ since $10 \times (25 + 1) = 10 \times 26 = 260$.
The final ordered pair is $(13, 19)$ since $13 \times (19 + 1) = 13 \times 20 = 260$.

The five ordered pairs (a, b) that satisfy the equation $a \times (b + 1) = 260$ as well as all the other conditions are $(2, 129)$, $(4, 64)$, $(5, 51)$, $(10, 25)$, and $(13, 19)$.