

Problem of the Week

Problem C and Solution

Round Round We Go

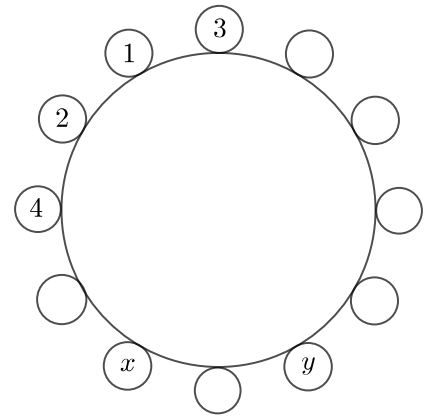
Problem

Each integer from 1 to 12 is to be placed around the outside of a circle so that the positive difference between any two integers next to each other is at most 2. The integers 3, 4, x , and y are placed as shown. What is the value of $x + y$?

Solution

Because two integers that are placed next to each other must have a difference of at most 2, then the possible neighbours of 1 are 2 and 3. Since 1 has exactly two neighbours, then 1 must be between 2 and 3.

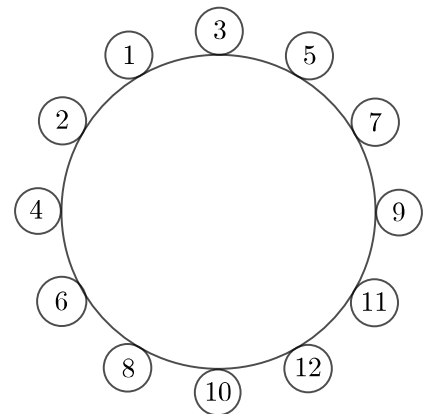
Next, consider 2. Its possible neighbours are 1, 3 and 4. The number 2 is already a neighbour of 1 and cannot be a neighbour of 3 (since 3 is on the other side of 1). Therefore, 2 is between 1 and 4. This allows us to update the diagram to the right.



Continuing in this way, the possible neighbours of 3 are 1, 2, 4, 5. The number 1 is already next to 3. Numbers 2 and 4 cannot be next to 3. So 5 must be next to 3.

The possible neighbours of 4 are 2, 3, 5, 6. The number 2 is already next to 4. Numbers 3 and 5 cannot be next to 4. So 6 must be next to 4.

Similarly, we know 7 will be next to 5 and 8 next to 6. This makes $x = 8$. Continuing this way, we know 9 is next to 7, 10 is next to 8, 11 is next to 9 and 12 is next to 10. This makes $y = 12$. This is shown in the completed circle to the right.



Therefore the sum of x and y is $8 + 12 = 20$.

