BEEP

Problem of the Week
Problem C and Solution
Wait for the Beep

Problem

The game *Beep* is played by a group of people counting up through the positive integers starting at 1. The first person says "one", the second "two", and so on. However, every time a multiple of 9, or a number containing the digit 9 is encountered, to avoid losing, the person must say "beep" instead of stating the number. For example, one part of the game would sound like this: "twelve", "thirteen", "fourteen", "fifteen", "sixteen", "seventeen", "beep", "beep", "twenty". "Eighteen" is replaced by "beep", since it is a multiple of 9, and "nineteen" is replaced by "beep", since it contains the digit 9.

If a group successfully makes it to 200, how many times has "beep" been said?

Solution

We first calculate the number of multiples of 9 between 1 and 200. Since $200 = (22 \times 9) + 2$, there are 22 multiples of 9 from 1 to 200, each of which will be replaced by one "beep".

Now let's look at how many of the integers from 1 to 200 contain the digit 9. These numbers are $9, 19, \ldots, 79, 89$ as well as $90, 91, \ldots, 98, 99$. There are 19 of these numbers from 1 to 100. There are another 19 between 100 and 200, which are obtained by adding 100 to the above numbers. Therefore, there are 38 integers from 1 to 200 that contain the digit 9, and 22 that are multiples of 9.

Some numbers that are multiples of 9 will also contain the digit 9, and will have been counted twice. There are 5 of these numbers between 1 and 200: 9, 90, 99, 189, and 198.

Hence, the number of positive integers less than 200 replaced by a "beep" is $38 + 22 - 5 = 55$, and the word "beep" is said 55 times.

EXTENSION: If a group successfully makes it to 2019, how many times has "beep" been said?