Problem of the Week
Problem C and Solution

The Biggest

Problem
$A, B, C, D, E$ and $F$ are each distinct nonzero digits, and

\[
\begin{array}{c}
A \ B \ C \\
+ \ D \ E \ F \\
\hline
1 \ 0 \ 0 \ 0
\end{array}
\]

Determine the largest possible three-digit integer $ABC$ that satisfies the restrictions above.

Solution
If we look at the ones column, since $C$ and $F$ are both digits from 1 to 9 and sum to a number that ends in 0, the sum must be 10. (The sum cannot be zero since neither $C$ nor $F$ is zero. The sum cannot be 20 or more, as $C$ and $F$ are digits.) That is, $C + F = 10$. Therefore, there is a carry of 1 into the tens columns. Similarly, the sum in the tens column must also be 10, so $B + E + 1 = 10$ or $B + E = 9$, and there is a carry of 1 into the hundreds column. Therefore, $A + D + 1 = 10$ or $A + D = 9$.

\[
\begin{array}{c}
1 \ 1 \\
A \ B \ C \\
+ \ D \ E \ F \\
\hline
1 \ 0 \ 0 \ 0
\end{array}
\]

To determine the largest value for $ABC$, $A$ must be as large as possible. Since $A + D = 9$, $A$ is largest when $A = 8$ and $D = 1$.
The next step is to make $B$ as large as possible. Since $B + E = 9$, then $B$ is largest when $B = 8$ and when $E = 1$. However, we already have $A = 8$ and $D = 1$ and we are not allowed repetitions. Therefore, the largest allowable value of $B$ occurs when $B = 7$ and $E = 2$.

Finally, we need to make $C$ as large as possible. Since $C + F = 10$, then $C$ is largest when $C = 9$ and $F = 1$. However, since we are not allowed repeated digits, we cannot use this solution. Also, we cannot use the solution $C = 8$ and $F = 2$, since we already have $A = 8$ and $E = 2$. We also cannot use the solution $C = 7$ and $F = 3$, since we already have $B = 7$. Therefore, the largest allowable value for $C$ is $C = 6$ when $F = 4$.

Therefore, the largest possible three-digit integer $ABC$ is 876.

Indeed, we can check that when $ABC$ is 876, we have $DEF$ equal to 124, and $ABC + DEF = 876 + 124 = 1000$. 