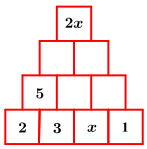


Problem of the Week
Problem C and Solution
Filtering Up

Problem

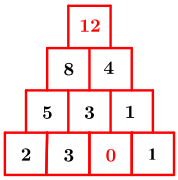
In the grid above, a number in any square located above the bottom row is obtained by adding the numbers connected to it from the row below. For example, the 5 in the second last row is obtained by adding the numbers connected to it in the row below, 2 and 3. The numbers filter up until reaching the final square containing $2x$. That is, the number in the top square is two times the unknown number in the bottom row. Determine the value of x.

Solution

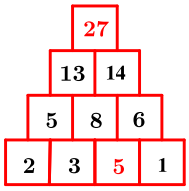
Solution 1

In the first solution a trial and error type solution will be presented. We will pick a value for x and then complete the grid.

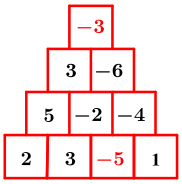
Let $x = 0$. We would expect $2x = 2(0) = 0$. However, after completing the grid, $2x = 12 \neq 0$. Therefore, $x \neq 0$.



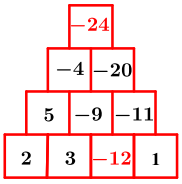
Let $x = 5$. We would expect $2x = 2(5) = 10$. After completing the grid, $2x = 27 \neq 10$. Therefore, $x \neq 5$. For our next trial we should choose a number lower than $x = 0$.



Let $x = -5$. We would expect $2x = 2(-5) = -10$. However, after completing the grid, $2x = -3 \neq -10$. Therefore, $x \neq -5$.



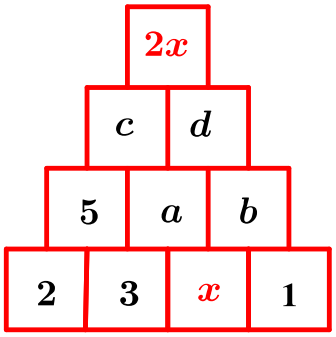
Let $x = -12$. We would expect $2x = 2(-12) = -24$. After completing the grid, $2x = -24$, the expected value. Therefore, $x = -12$.



This solution is valid but not efficient. To reach the solution could take many, many trials. In the second solution, we will look at an algebraic approach.

Solution 2

For easy reference, label all of the empty, unshaded squares in the grid as shown.



We can complete the second row using the addition rule for the grid, $a = 3 + x$ and $b = x + 1$.

Moving to the third row, $c = 5 + a = 5 + 3 + x = 8 + x$ and $d = a + b = 3 + x + x + 1 = 4 + 2x$.

Finally, in the fourth row, $2x = c + d = 8 + x + 4 + 2x = 3x + 12$.

We can now solve the equation.

$$\begin{aligned} 2x &= 3x + 12 & \\ 2x - 2x &= 3x - 2x + 12 & \text{Subtracting } 2x \text{ from both sides} \\ 0 &= x + 12 & \text{Simplifying} \\ 0 - 12 &= x + 12 - 12 & \text{Subtracting 12 from both sides} \\ x &= -12 & \end{aligned}$$

Therefore, $x = -12$.

An algebraic solution to this problem is much more efficient. Some students may not quite have the necessary background to complete this solution on their own at this point.