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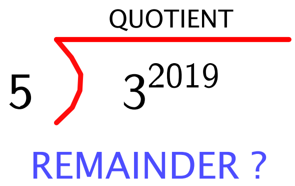

Problem of the Week

Problem C and Solution

What Remains?

Problem

3^5 means $3 \times 3 \times 3 \times 3 \times 3$ and equals 243 when expressed as an integer. When 3^5 is divided by 5, the remainder is 3. When the integer 3^{2019} is divided by 5, what is the remainder?

Solution

When the units digit of an integer is 0 or 5, the remainder is 0 when divided by 5. When the units digit of an integer is 1 or 6, the remainder is 1 when divided by 5. For example, $56 \div 5 = 11$ R1. When the units digit of an integer is 2 or 7, the remainder is 2 when divided by 5. When the units digit of an integer is 3 or 8, the remainder is 3 when divided by 5. When the units digit of an integer is 4 or 9, the remainder is 4 when divided by 5.

So let's examine the pattern of the units digits on the first eight powers of 3.

Exponent	Power	Value	Units Digit
1	3^1	3	3
2	3^2	9	9
3	3^3	27	7
4	3^4	81	1
5	3^5	243	3
6	3^6	729	9
7	3^7	2187	7
8	3^8	6561	1

It would appear that there is a pattern in the units digits that repeats every four powers of 3. The pattern would continue: the units digits of 3^9, 3^{10}, 3^{11}, 3^{12} would be 3, 9, 7, 1, respectively. So we need to determine how many complete groups of four are in 2019. We determine that $2019 \div 4 = 504$ R3. There are 504 complete repetitions of the pattern of units digits and 3 numbers into the next pattern. This means that the units digit of 3^{2019} is 7, the third number in the pattern of the units digits. Since the units digit is 7, the remainder when 3^{2019} is divided by 5 is 2.

$\therefore$ when 3^{2019} is divided by 5, the remainder is 2.