Problem of the Week
Problem C and Solution
Lookover Lane

Problem

Lookover Lane divides two rows of houses. Each house on one side of the lane is directly opposite a house on the other side of the lane. The houses are numbered consecutively $1, 2, 3$, and so on along one side. Once the end of that side of the lane is reached, the consecutive numbering continues at the house on the other side of the street opposite house number 1. The consecutive numbering continues along this second side until the last house is numbered. When the residents of house number 37 look directly across the lane, they see the house numbered 84. How many houses are on Lookover Lane?

Solution

Solution 1

In this solution we will use logic to reason the number of houses on Lookover Lane.

There are 36 houses before house 37 on the one side of the lane. Therefore, there must be 36 houses on the other side of the lane before house number 84.

So, the first house on the other side of the lane is house number $84 - 36 = 48$. Therefore, the last house on the first side of the lane is house number 47.

Each house on one side has a house directly across from it on the other side. Since there are 47 houses on one side, there are 47 houses on the other side and there are a total of $47 \times 2 = 94$ houses on Lookover Lane.

Solution 2

In this solution, a variable is introduced to help in the argument.

Suppose there are n houses on one side of Lookover Lane. Then there are a total of $2 \times n = 2n$ houses on both sides of the lane. House 1 is opposite house $n + 1$, house 2 is opposite house $n + 2$, house 3 is opposite house $n + 3$, and so on. (The ... below represent the houses in between house 3 and house 37 and again the houses between house 37 and house n. Similarly for the second row) House 37 is opposite house 84 and house n is opposite house $2n$.

$$
\begin{array}{ccccccc}
1 & 2 & 3 & \ldots & 37 & \ldots & n \\
n+1 & n+2 & n+3 & \ldots & 84 & \ldots & 2n
\end{array}
$$

There are 36 houses before house 37. Therefore, there must be 36 houses before house 84.

So, the first house in the second row is house number $84 - 36 = 48$. This is house $n + 1$. Therefore, the last house in the first row, house n, is house number 47.

Therefore, there are $47 \times 2 = 94$ houses on Lookover Lane.