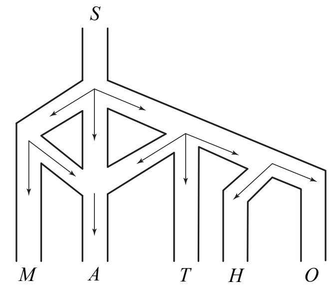
Problem of the Week

Problem D and Solution

Marble Maze

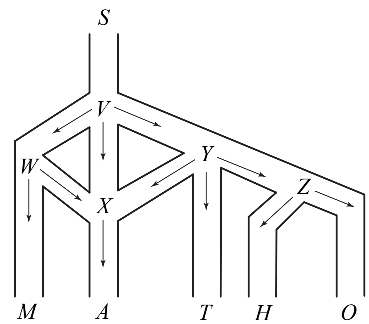
Problem



In **MATHO**, the game described below, what is the probability of winning?

In the game **MATHO**, a marble is dropped into the maze at S. The marble will move down through the maze in the direction of the arrows. At any junction, the marble is equally likely to go in any of the possible downward paths. If the marble ends up in A, you lose. Otherwise, you win.

Solution



We label the five junctions as V, W, X, Y, and Z.

We will calculate the probability of losing **MATHO**. The probability of winning can then be obtained by subtracting the probability of losing from 1. You lose the game if your marble ends up at A. From the arrows the marble can follow, we see that in order to get to A, the marble must go through X (from X the marble must go to A). So we will calculate the probability that the marble goes to X.

To get to X, the marble can go from S to V to W to X, or S to V to X, or S to V to Y to X.

At V, the probability that the marble goes down any of the three paths (that is, towards W, X or Y) is $\frac{1}{3}$. So the probability that the marble goes directly from V to X is $\frac{1}{3}$.

At W, the probability that the marble turns to X is $\frac{1}{2}$, so the probability that the marble goes from V to W and from W to X is $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$.

At Y, the probability that the marble turns to X is $\frac{1}{3}$, so the probability that the marble goes from V to Y and Y to X is $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

Therefore, the probability that the marble gets to X (and thus ends up at A) is $\frac{1}{3} + \frac{1}{6} + \frac{1}{9} = \frac{11}{18}$. That is, the probability of losing the game is $\frac{11}{18}$. Therefore, the probability of winning the game is $1 - \frac{11}{18} = \frac{7}{18}$, or approximately 38.9%.

Extension: A *fair game* is a game in which there is an equal chance of winning or losing. If a game is fair, then we can say that the probability of winning is equal to the probability of losing. **MATHO** is not a fair game. Can you create a modified version of **MATHO** that is fair?

A Final Thought: Notice that there are 3 different paths from S to A ($SVWXA$, $SVXA$, $SVYXA$) and the total number of possible paths that the marble could take is 7 (check this for yourself!). You might be tempted to conclude that the probability of the marble ending up at A is $\frac{3}{7}$. This is not the case. You would be incorrectly assuming that the marble is equally likely to take each path. We are given that at any junction, the marble is equally likely to go in any of the downward paths. We are not told that each path is equally likely. In fact, each path is not equally likely, which is why we cannot determine the probability this way.