



Problem of the Week

Problem D and Solution

New Heights

Problem

The heights of the ten members of Ricky's basketball team were measured. The heights, in cm, of his nine teammates are 180, 181, 183, 187, 188, 190, 193, 195, and 196. Ricky's height is also a whole number. The coach of the team made a mistake when measuring Ricky's height. After this mistake was corrected, both the mean and median of the heights increased by 0.5 cm. Find all possible correct values for Ricky's height.

Solution

We will let Ricky's correct height be R and the incorrectly recorded height be r . To find a relationship between R and r we will look at the averages of the heights. The average of the heights with the incorrect height, r , is

$$\frac{180+181+183+187+188+190+193+195+196+r}{10} = \frac{1693+r}{10}.$$

The average with the correct height, R , is

$$\frac{180+181+183+187+188+190+193+195+196+R}{10} = \frac{1693+R}{10}.$$

Therefore,

$$\begin{aligned} \frac{1693+r}{10} + 0.5 &= \frac{1693+R}{10} \\ 1693+r+5 &= 1693+R \\ r+5 &= R \end{aligned}$$

Let's look at the median for different values of r .

Case 1: If $r \leq 182$, then $R \leq 187$ (since $R = r + 5$).

The median with the incorrect height is $\frac{187+188}{2} = 187.5$. Since R and r are both less than 188, the median with the correct height will remain at 187.5. Therefore, this case is not a possible solution.

Case 2: If $r = 183$, then $R = 188$.

The median with the incorrect height is $\frac{187+188}{2} = 187.5$. Since $R = 188$, the median with the correct height is $\frac{188+188}{2} = 188$, which is an increase of 0.5 in the median. Therefore, this case is one possible solution.

Case 3: If $r = 184$, then $R = 189$.

The median with the incorrect height is $\frac{187+188}{2} = 187.5$. Since $R = 189$, the median with the correct height is $\frac{188+189}{2} = 188.5$, which is an increase of 1 in the median. Therefore, this case is not a possible solution.





Case 4: If $185 \leq r \leq 187$, then $190 \leq R \leq 192$.

The median with the incorrect height is $\frac{187+188}{2} = 187.5$. Since $190 \leq R \leq 193$, the median with the correct height is $\frac{188+190}{2} = 189$, which is an increase of 1.5 in the median. Therefore, this case is not a possible solution.

Case 5: If $r = 188$, then $R = 193$.

The median with the incorrect height is $\frac{188+188}{2} = 188$. Since $R = 193$, the median with the correct height is $\frac{188+190}{2} = 189$, which is an increase of 1 in the median. Therefore, this case is not a possible solution.

Case 6: If $r = 189$, then $R = 194$.

The median with the incorrect height is $\frac{188+189}{2} = 188.5$. Since $R = 194$, the median with the correct height is $\frac{188+190}{2} = 189$, which is an increase of 0.5 in the median. Therefore, this case is one possible solution.

Case 7: If $r \geq 190$, then $R \geq 195$.

The median with the incorrect height is $\frac{188+190}{2} = 189$. Since both r and R are greater than or equal to 190 the median with the correct height will remain at $\frac{188+190}{2} = 189$. Therefore, this case is not a possible solution.

Therefore, the possible correct heights for Riky are 188 cm or 194 cm.

