



Problem of the Week

Problem D and Solution

Get Your Tickets



Problem

A box contains a total of 400 tickets that come in five colours: blue, green, red, yellow and orange. The ratio of blue to green to red tickets is $1 : 2 : 4$. The ratio of green to yellow to orange tickets is $1 : 3 : 6$. How many tickets are there of each colour?

Solution

Solution 1

We denote the number of tickets of each of the five colours by the first letter of the colour. We are given that $b : g : r = 1 : 2 : 4$ and that $g : y : o = 1 : 3 : 6$.

Through multiplication by 2, the ratio $1 : 3 : 6$ is equivalent to the ratio $2 : 6 : 12$. Thus, $g : y : o = 2 : 6 : 12$.

We chose to scale this ratio by a factor of 2 so that the only colour common to the two given ratios, green, now has the same number in both of these ratios.

That is, $b : g : r = 1 : 2 : 4$ and $g : y : o = 2 : 6 : 12$ and since the term g is 2 in each ratio, then we can combine these to form a single ratio,

$$b : g : r : y : o = 1 : 2 : 4 : 6 : 12$$

This ratio tells us that for every blue ticket, there are 2 green, 4 red, 6 yellow, and 12 orange tickets. Thus, if there was only 1 blue ticket, then there would be $1 + 2 + 4 + 6 + 12 = 25$ tickets in total.

However, we are given that the box contains 400 tickets in total. Therefore, the number of blue tickets in the box is $\frac{400}{25} = 16$.

Through multiplication by 16, the ratio $b : g : r : y : o$ becomes $16 : 32 : 64 : 96 : 192$.

Therefore, there are 16 blue, 32 green, 64 red, 96 yellow, and 192 orange tickets. (Note that there are $16 + 32 + 64 + 96 + 192 = 400$ tickets in total.)





Solution 2

We denote the number of tickets of each of the five colours by the first letter of the colour. We are given that $b : g : r = 1 : 2 : 4$ and that $g : y : o = 1 : 3 : 6$.

The first ratio tells us that $\frac{b}{g} = \frac{1}{2}$, and so $b = \frac{g}{2}$.

The first ratio also tells us that $\frac{g}{r} = \frac{2}{4}$, and so $r = 2g$.

The second ratio tells us that $\frac{g}{y} = \frac{1}{3}$, and so $y = 3g$.

The second ratio also tells us that $\frac{g}{o} = \frac{1}{6}$, and so $o = 6g$.

We are given that there are a total of 400 tickets. That is, $b + g + r + y + o = 400$.

Substituting $b = \frac{g}{2}$, $r = 2g$, $y = 3g$, and $o = 6g$, this becomes

$$\frac{g}{2} + g + 2g + 3g + 6g = 400$$

$$\frac{25}{2}g = 400$$

$$g = 32$$

Thus, $b = \frac{g}{2} = 16$, $r = 2g = 64$, $y = 3g = 96$, and $o = 6g = 192$.

Therefore, there are 16 blue, 32 green, 64 red, 96 yellow, and 192 orange tickets.

