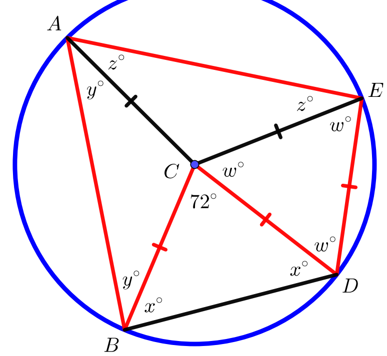
Problem of the Week
Problem D and Solution
More Angle Chasing

Problem

The points A, B, D and E lie on the circumference of a circle with centre C. If $\angle BCD = 72^\circ$ and $CD = DE$, determine the measure of $\angle BAE$.

Solution



As the solution proceeds, the new markings on the diagram will be explained. Draw radii from C to points A and E on the circumference. Join B to D.

Since CB and CD are radii, $CB = CD$ and $\triangle CBD$ is isosceles. Therefore, $\angle CBD = \angle CDB = x^\circ$. But in a triangle the angles sum to 180°. It follows that $x^\circ + x^\circ + 72^\circ = 180^\circ$. Then $2x^\circ = 108^\circ$ and $x = 54$.

In $\triangle CDE$, $CD = CE$ since they are both radii. But we are given that $CD = DE$. Therefore, $CD = CE = DE$ and $\triangle CDE$ is equilateral. It follows that each angle is 60°. Therefore, $w = 60$.

Since CE and CA are radii, $CE = CA$ and $\triangle CEA$ is isosceles. Therefore, $\angle CEA = \angle CAE = z^\circ$.

Similarly, since CA and CB are radii, $CA = CB$ and $\triangle CAB$ is isosceles. Therefore, $\angle CAB = \angle CBA = y^\circ$.

The figure $ABDE$ is a quadrilateral and we know that the sum of the interior angles of a quadrilateral is 360°. Then,

$$\begin{aligned}
\angle BAE + \angle ABD + \angle BDE + \angle DEA &= 360^\circ \\
(y^\circ + z^\circ) + (y^\circ + x^\circ) + (x^\circ + w^\circ) + (w^\circ + z^\circ) &= 360^\circ \\
2w^\circ + 2x^\circ + 2y^\circ + 2z^\circ &= 360^\circ \\
w^\circ + x^\circ + y^\circ + z^\circ &= 180^\circ \qquad \text{Dividing by 2} \\
(60^\circ) + (54^\circ) + y^\circ + z^\circ &= 180^\circ \qquad \text{Substituting for } w \text{ and } x \\
(114^\circ) + y^\circ + z^\circ &= 180^\circ \\
(y+z)^\circ &= 66^\circ \\
\angle BAE &= 66^\circ \qquad \text{Since } \angle BAE = (y+z)^\circ
\end{aligned}$$

$\therefore \angle BAE = 66^\circ$.