



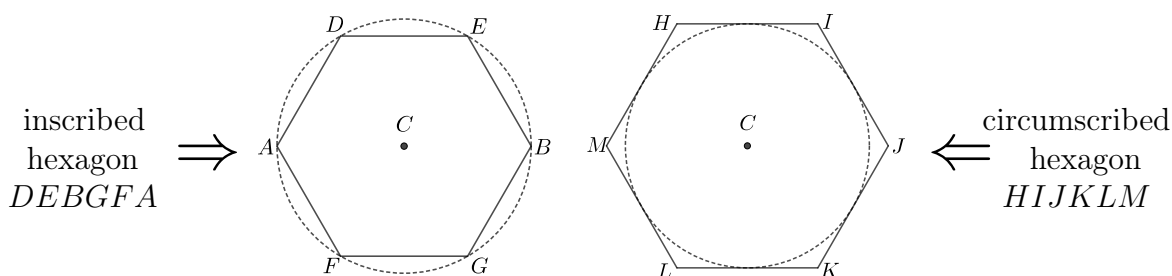
Problem of the Week

Problem D

Pi Day Hexagons

Pi Day is an annual celebration of the mathematical constant π . Pi Day is observed on March 14 since 3, 1, and 4 are the first three significant digits of π .

Archimedes determined lower and upper bounds for π by finding the perimeters of regular polygons inscribed and circumscribed in a circle with a diameter of length 1. (An inscribed polygon of a circle has all vertices on the circle. A circumscribed polygon of a circle has all sides tangent to the circle.) We will determine such bounds by looking at regular hexagons inscribed and circumscribed in a circle with centre C and diameter 1.

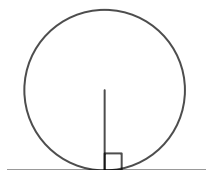
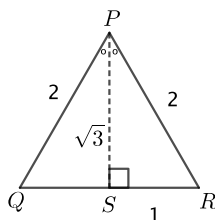


Since the circle has circumference equal to π , the perimeter of the inscribed regular hexagon $DEBGF A$ will give a lower bound for π and the perimeter of the circumscribed regular hexagon $HIJKLM$ will give an upper bound for π .

Using these hexagons, determine a lower and an upper bound for π .

Some may find the following facts to be useful:

1. When you drop a perpendicular from a vertex of an equilateral triangle to the opposite side, you bisect the angle and the third side. So if we let one side of the equilateral triangle have length 2, we get the following information (below left).



2. The radius of a circle is perpendicular to a tangent of the circle at the point of tangency (above right).
3. The centres of both the inscribed and circumscribed regular hexagons will be at the centre of the circle, C .

