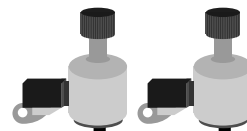


## Problem of the Week

### Problem D and Solution

#### So Many Dynamos



#### Problem

A palindrome is a word, phrase, sentence, or number that reads the same forwards and backwards. Determine the number of six-digit palindromic numbers which are divisible by 15.

#### Solution

We are looking for a six-digit number of the form  $abccba$ .

For a number to be divisible by 15, it must be divisible by both 3 and 5.

To be divisible by 5, a number must end in 0 or 5. If the required number ends in 0, it must also begin with 0 in order to be a palindrome. But the number  $0bccb0$  is not a six-digit number. Therefore, the number cannot end in a 0 and hence must start and end with a 5. The required number looks like  $5bccb5$ .

For a number to be divisible by 3, the sum of its digits must be divisible by 3. Therefore, we get the sum  $10 + 2b + 2c$  must be divisible by 3.

Since  $b$  and  $c$  are digits, we note that  $0 \leq b, c \leq 9$ , where  $b$  and  $c$  are integers. This leads to  $10 \leq 10 + 2b + 2c \leq 46$ .

The numbers between 10 and 46 that are divisible by 3 are 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, and 45.

If we choose 12 to be the sum of the digits we get the equation  $10 + 2b + 2c = 12$ . We can simplify this equation to an equivalent equation.

$$\begin{aligned}10 + 2b + 2c &= 12 \\2(b + c) &= 2 \\b + c &= 1\end{aligned}$$

This equivalent equation has the solutions  $b = 1, c = 0$  or  $b = 0, c = 1$ .

If we choose 15 to be the sum of the digits we get the equation  $10 + 2b + 2c = 15$ .

We can simplify this equation to an equivalent equation.

$$\begin{aligned}10 + 2b + 2c &= 15 \\2(b + c) &= 5 \\b + c &= 2.5\end{aligned}$$

Since  $b$  and  $c$  are integers, this equation has no solution. Similarly, for any of the odd sums there will no solutions.





Therefore, we only need to solve the equations with even sums.

We will summarize the results in a table.

| Original Equation   | Equivalent Equation | Solutions in the form $(b, c)$                                  | Number of Solutions |
|---------------------|---------------------|---|---------------------|
| $10 + 2b + 2c = 12$ | $b + c = 1$         | $(0,1), (1,0)$  | 2                   |
| $10 + 2b + 2c = 18$ | $b + c = 4$         | $(0,4), (1,3), (2,2), (3,1), (4,0)$                             | 5                   |
| $10 + 2b + 2c = 24$ | $b + c = 7$         | $(0,7), (1,6), (2,5), (3,4), (4,3), (5,2), (6,1), (7,0)$        | 8                   |
| $10 + 2b + 2c = 30$ | $b + c = 10$        | $(1,9), (2,8), (3,7), (4,6), (5,5), (6,4), (7,3), (8,2), (9,1)$ | 9                   |
| $10 + 2b + 2c = 36$ | $b + c = 13$        | $(4,9), (5,8), (6,7), (7,6), (8,5), (9,4)$                      | 6                   |
| $10 + 2b + 2c = 42$ | $b + c = 16$        | $(7,9), (8,8), (9,7)$   | 3                   |

Therefore, the total number of solutions is  $2 + 5 + 8 + 9 + 6 + 3 = 33$ .

