



## Problem of the Week

### Problem D and Solution

#### Multiply then Add

#### Problem

The product of two integers is computed. The smaller integer is then added to the product, resulting in the sum 299. Find the four integer pairs that satisfy the conditions of the problem.

#### Solution

Let  $x$  represent the smaller integer.

Let  $y$  represent the larger integer. We know that  $y > x$ .

The product of the two integers is  $xy$ . To this product we add the smaller integer  $x$ . The resulting expression is  $xy + x$ . We want to find all possible pairs of integers satisfying the equation  $xy + x = 299$ .

Factoring the left side of the equation, we obtain  $x(y + 1) = 299$ . We want the product of two integers to be 299. Either both integers are positive or both integers are negative. The factors of 299 are  $\{\pm 1, \pm 13, \pm 23, \pm 299\}$ .

Then  $299 = (-299) \times (-1) = (-23) \times (-13) = 1 \times 299 = 13 \times 23$ .

The first integer in each product is the smaller of the two integers and is therefore the value of  $x$ . The second integer in each pair is larger and corresponds to  $y + 1$ . So, the value of  $y$  is 1 less than the second integer in each of the products.

Therefore, the ordered pairs satisfying  $xy + x = 299$  are  $(x, y) = (-299, -2)$ ,  $(x, y) = (-23, -14)$ ,  $(x, y) = (1, 298)$ , and  $(x, y) = (13, 22)$ .

We can check the validity of each pair by substituting into the expression  $xy + x$  to confirm that the value is 299.

When  $x = -299$  and  $y = -2$ ,

$$xy + x = (-299)(-2) + (-299) = 598 - 299 = 299.$$

When  $x = -23$  and  $y = -14$ ,  $xy + x = (-23)(-14) + (-23) = 322 - 23 = 299$ .

When  $x = 1$  and  $y = 298$ ,  $xy + x = (1)(298) + (1) = 298 + 1 = 299$ .

When  $x = 13$  and  $y = 22$ ,  $xy + x = (13)(22) + (13) = 286 + 13 = 299$ .

