



Problem of the Week

Problem D and Solution

To Top It Off

Problem

Jen, John and Rob ordered two medium pizzas. Each of the medium pizzas is divided into 12 equal sized slices. Each slice can have at most 3 toppings and any two slices of the same pizza may have different toppings. The available toppings are pepperoni, green pepper, mushrooms, onion and pineapple. Their pizzas were divided as follows:

- Jen ate $\frac{1}{4}$ of all the slices. All of her slices had pepperoni as a topping, two of her slices had mushrooms and two had pineapple.
- John ate $\frac{1}{3}$ of all the slices. All of his slices had mushrooms as a topping, four of his slices also had green pepper and onions.
- Rob ate all of the remaining slices. Four of his slices had only two toppings, onion and pineapple. His remaining slices each had exactly 3 toppings.

If $\frac{1}{2}$ of the slices contained pepperoni, $\frac{1}{4}$ of the slices contained pineapple and $\frac{1}{6}$ of the slices had green pepper, determine the toppings on Rob's remaining slices.

Solution

Jen, John and Rob ordered two pizzas, each with 12 slices. Therefore, they have a total of $2 \times 12 = 24$ slices of pizza.

Jen ate $\frac{1}{4}$ of the total number of slices, so she ate $\frac{1}{4} \times 24 = 6$ slices of pizza. All 6 slices had pepperoni, two also had mushrooms and two also had pineapple.

John ate $\frac{1}{3}$ of the total number of slices, so he ate $\frac{1}{3} \times 24 = 8$ slices of pizza. All 8 slices had mushrooms, four also had green pepper and onions. So four of the slices had mushrooms, green pepper and onions, and 4 had at least mushrooms.

Rob ate all of the remaining slices, so he ate $24 - 6 - 8 = 10$ slices. Four of these slices only had onion and pineapple. We are asked to determine the toppings on Rob's remaining 6 slices.

We are given that $\frac{1}{2}$ of the total number of slices were covered in pepperoni, so $\frac{1}{2} \times 24 = 12$ slices of pizza had pepperoni. Jen ate 6 of these 12 slices. Therefore, there were 6 other slices with pepperoni on them.

We are also given that $\frac{1}{4}$ of the total number of slices were covered in pineapple, so $\frac{1}{4} \times 24 = 6$ slices of pizza had pineapple. Rob ate 4 of these 6 slices and Jen ate 2. Therefore, there were no other slices with pineapple on them.

Further, we are given that $\frac{1}{6}$ of the total number of slices were covered in green pepper, so $\frac{1}{6} \times 24 = 4$ slices of pizza had green pepper. John ate 4 of these slices. Therefore, there were no other slices with green pepper on them.

Of the five toppings available, pepperoni, green pepper, mushrooms, onion and pineapple, the remaining six slices that Rob ate could not have pineapple or green pepper on them. Since we are also given that these six slices have exactly 3 toppings on them, they must have the toppings pepperoni, mushrooms and onions.

Therefore, Rob's remaining slices all have the same three toppings: pepperoni, mushrooms and onions.

