Problem of the Week

Problem D and Solution

It's a New Year

Problem

5^3 is a *power* with *base* 5 and *exponent* 3. 5^3 means $5 \times 5 \times 5$ and equals 125 when expressed as an integer. When $8^{672} \times 5^{2019}$ is expressed as an integer, how many digits are in the product?

Solution

An immediate temptation might be to reach for a calculator. In this case, basic calculator technology will let you down. We will look at the problem using our knowledge of powers and corresponding power laws.

$$\begin{aligned}
8^{672} \times 5^{2019} &= \left((2^3)^{672}\right) \times 5^{2019} \\
&= 2^{3\times 672} \times 5^{2019} \\
&= 2^{2016} \times 5^{2019} \\
&= 2^{2016} \times 5^{2016} \times 5^3 \\
&= (2 \times 5)^{2016} \times 125 \\
&= 10^{2016} \times 125
\end{aligned}$$

But 10^{2016} is the number 1 followed by 2016 zeroes. When we multiply this number by the three-digit number 125, we obtain the number 125 followed by 2016 zeroes. Therefore, $8^{672} \times 5^{2019}$ has $2016 + 3 = 2019$ digits. Happy New Year again!