



## Problem of the Week

### Problem D and Solution

#### Rotten Fruit



#### Problem

At Math's Grocers Alan stocks apples and Pruneet stocks pears. One day they noticed that an equal number of apples and pears were rotten. Also,  $\frac{2}{3}$  of the apples were rotten and  $\frac{3}{4}$  of the pears were rotten. Of the total number of apples and pears, what fraction was rotten?

#### Solution

##### Solution 1

Let the total number of apples be represented by  $a$  and the total number of pears be represented by  $p$ . Since there were an equal number of rotten apples and rotten pears, then  $\frac{2}{3}a = \frac{3}{4}p$ , so  $p = \frac{4}{3}(\frac{2}{3}a) = \frac{8}{9}a$ .

Therefore, the total number of apples and pears was  $a + p = a + \frac{8}{9}a = \frac{17}{9}a$ .

Also, the total number of rotten fruit was  $2(\frac{2}{3}a) = \frac{4}{3}a$ .

Therefore, the fraction of the total amount of fruit that was rotten was

$$\frac{\frac{4}{3}a}{\frac{17}{9}a} = \frac{4}{3} \left( \frac{9}{17} \right) = \frac{12}{17}.$$

##### Solution 2

Since  $\frac{2}{3}$  of the the apples were rotten,  $\frac{3}{4}$  of the pears were rotten, and the number of rotten apples equaled the number of rotten pears, then we could let the number of rotten apples be 6. The number of rotten pears will also be 6. (We choose 6 as it is a multiple of the numerator of each fraction.)

If there were 6 rotten apples, then the total number of apples is

$$6 \div \frac{2}{3} = 6\left(\frac{3}{2}\right) = 9.$$

If there were 6 rotten pears, then the total number of pears is

$$6 \div \frac{3}{4} = 6\left(\frac{4}{3}\right) = 8.$$

Therefore, there were  $9 + 8 = 17$  pieces of fruit in total, of which  $6 + 6 = 12$  were rotten.

Thus,  $\frac{12}{17}$  of the fruit was rotten.

NOTE: In solution 2 we could have used any multiple 6 for each of the number of rotten apples and the number of rotten pears. The final fraction would eventually reduce to  $\frac{12}{17}$ .

